2. Simulation and Optimization of MEMS
2.3 Nonlinear macromodeling

- The modeling level described as *macromodeling* plays several roles in the overall design scheme.
- First, consider that a designer might want to use a macromodel to explore a design space, that is, to predict easily how behavior will change as dimensions or material properties are changed.
- Second, the macromodel will represent the MEMS device in a system-level simulator. Therefore, the macromodel must be *dynamic*, and must be simple enough to permit hundreds or thousands of dynamical simulations under a variety of excitations in reasonable time.
Third, because MEMS devices are usually transducers involving multiple energy domains, the macromodel should correctly account both for energy conservation (a quasi-static property), and energy dissipation (a dynamic property).

Finally, the macromodel should agree with the results of more detailed numerical simulation over some design space of interest, and should be based on approximations that have been compared carefully with experiment on suitable designed test structures.
Creating quasi-static macromodels starting from simplified analytical formulations. The procedure is roughly as follows:

1. Select an idealized structure that is close to the desired model.
2. Model the idealized problem analytically, either by solving the governing differential equation, or by approximating the solution with Rayleigh-Ritz energy minimization methods.
(3) Identify a set of nondimensionalized numerical constants that can be varied within the analytical form of the solution.

(4) Perform meshed numerical simulations of the desired structure over the design space of interest, and adjust the nondimensionalized numerical quantities in the macromodel for agreement with the numerical simulations.
Membrane load-deflection

As an example, consider the pressure-deflection behaviour of a thin elastic membrane, suspended on rigid frame (Fig. 12)

Figure 12. Suspended membrane illustrating load-deflection behavior under a uniform pressure load.
If a differential pressure is applied to this structure, it deforms out of plane.

The center-deflection $d$ is related to the applied pressure $P$ through an equation that depends on the geometry of the membrane and the material constants (Young's modulus $E$, Poisson ratio $\nu$, and residual stress $\sigma$).

If the membrane is modeled as a pure membrane (with no bending stiffness), closed-form analytical solutions can be found for circular geometries, and both power-series and Rayleigh-Ritz approximants can be found for square and rectangular geometries.
In this case, the Rayleigh-Ritz form is quite helpful, because it yields closed-form expressions with explicit dimensions and material constants.

The resulting form of the pressure-deflection relation, including the added nondimensionalized adjustable parameters, is expressed as

\[
P = \frac{C_1 t}{a^2} \sigma d + \frac{C_2 f(v) t}{a^4} \frac{E}{1 - \nu} d^3, \tag{16}
\]

where \(a\) is the radius/half-edge length of the membrane and \(t\) is the thickness.
The dimensionless constants $C_1$ and $C_2$ and the dimensionless function $f(\nu)$ are determined from fitting Equation 16 to the results of extensive finite-element simulations over a range of dimensions (length and thickness) and material constants.

Note that a factor $(1-\nu)$ appears in the denominator, with the rest of the $\nu$-dependence captured by $f(\nu)$, which is a slowly varying function, changing only 6 percent for a variation of $\nu$ over the physically interesting range 0.3-0.5.
Numerical constants obtained for variously shaped membranes are provided in Table 2, in which “rectangular” refers to rectangular shapes with a width-to-length ratio of greater than 8:1.

<table>
<thead>
<tr>
<th>Membrane shape</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$f(v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>4.00</td>
<td>2.67</td>
<td>$0.957 - 0.208v$</td>
</tr>
<tr>
<td>Square</td>
<td>3.41</td>
<td>1.37</td>
<td>$1.446 - 0.427v$</td>
</tr>
<tr>
<td>Rectangular</td>
<td>2.00</td>
<td>1.33</td>
<td>$(1 + v)^{-1}$</td>
</tr>
</tbody>
</table>
The load-deflection behavior captured in Eq. 16 using the constants in Table 2 has been very successfully applied to a wide variety of microfabricated structures, both for prediction of performance and for the extraction of material constants from test devices.
Electrostatic pull-in of beams

- This same method can be used to analyze the instability point of an electrostatically actuated elastic structure.

- Figure 13 illustrates a conducting beam of thickness $t$, length $L$, and width $W$, clamped at both ends by dielectric supports, and suspended over a ground plane by a gap $g_0$.

- When a voltage is applied between the beam and the ground plane, the charges on the beam and the ground plane produce an attractive force between them, which causes the beam to bend toward the ground plane.
Figure 13

Electrostatically actuated fixed-fixed beam. When $V = V_p$, the beam collapses.
Figure 14 shows the normalized dependence of the deflection of the center of the beam on applied voltage.
This pull-in event is sharp, and readily observed experimentally.

It depends strongly both on the beam's geometry and on its material constants.

This has made possible the development of a method for material-property measurement that is called M-Test.

The models for the dependence of pull-in voltage on geometry and material properties within M-Test where derived initially from analytical beam theory, with three adjustable constants added for extensive fitting to the results of fully meshed coupled-energy domain simulations.
Dynamical macromodels are much more challenging than quasi-static ones, particularly when the design space involves large motions and nonlinear forces.

Explicit dynamical formulations for meshed structures have many drawbacks.

First, they are typically relatively slow.
Second, being fully numerical, explicit dynamical simulations of the designer to probe sensitivities to variations in geometry or material constants without simply performing multiple analyses and building up a large database.

Third, fully meshes models are too computationally expensive to insert in system-level simulators.
For all these reasons, we have to seek methods which permit projection of the results of fully-meshed analyses onto physically meaningful reduced variables sets, preferably already containing the appropriate algebraic dependences on structural dimensions and material constants.
References
