Stop & Go Vehicle Motion Model
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Abstract – In this paper a longitudinal vehicle model targeted towards low velocity adaptive cruise control systems is presented.

I. INTRODUCTION

The automotive industry has been encouraging the development of advanced concepts that may lead to more efficient traffic flow. Already various driver-assistance systems including lane-keeping and distance-keeping have been successfully developed and are being commercialised. Conventional adaptive cruise control systems have been available for more than ten years [1]. They operate at speeds above 40km/h by controlling the throttle input. However, they do not cope successfully with low speed travel due to deficiencies in the adaptive control algorithms used.

Advanced adaptive control concepts are therefore required to facilitate the development of a slow-motion adaptive-cruise-control system, i.e. Stop & Go Adaptive Cruise Controller (Stop & Go ACC) [2].

In this report, a longitudinal vehicle motion model for Stop & Go ACC is discussed.

II. OUTLINE OF STOP & GO ACC

The goal of Stop & Go ACC is to assist human drivers whilst in traffic jams by reducing the need for them to repeatedly accelerate and/or stop their vehicles. The basic configuration of a Stop & Go ACC is shown in Fig.1. An outline of the differences between conventional ACC systems and Stop & Go systems is summarized in Table 1 [2].

The majority of published adaptive cruise control models are related to high speed motion. Engine dynamics and brake system characteristics under slow motion conditions are usually disregarded.

<table>
<thead>
<tr>
<th>Condition of operation</th>
<th>ACC</th>
<th>Stop &amp; Go</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition of operation</td>
<td>Assists between 40 ± 10km/h and 170 ± 10 km/h by action on accelerator and brakes</td>
<td>Assists between 0 km/h and 40 ± 10 km/h by action on accelerator and brakes</td>
</tr>
<tr>
<td>Preceding vehicle selection</td>
<td>Moving vehicle in the driving path of equipped vehicle</td>
<td>Moving and stopped vehicles in the driving path of equipped vehicle</td>
</tr>
<tr>
<td>Maximum braking capacity</td>
<td>Limited braking capacity (from –1.5 to –3.5 m/s², depending on the system)</td>
<td>Automatic braking capacity enhanced to 4m/s², or even –5m/s²</td>
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</tbody>
</table>

Table 1. Outline of the difference between ACC and Stop & Go systems

III. Longitudinal Vehicle Model

Most vehicles currently on the road have an internal combustion engine as their source of power. Their mathematical models have frequently been examined [3]. At high speeds (> 40 km/h), a torque converter is mechanically locked and the gear ratio is not changed sharply. The longitudinal vehicle model can be remarkably simplified by taking these factors into consideration. Unfortunately these assumptions are not fulfilled under low speed conditions.

In this work we make the following assumptions:

1. no torsion of the drive axle
2. no slip at the wheels

Based on the free-body diagram of a rear-wheel drive vehicle (Fig.2) we assume a bicycle model for force balance. This leads to the equation [4].

\[ ma = F_x + F_y - F_d \]  

where \( F_x \) and \( F_y \) are the traction forces and \( F_d \) is the drag force on the vehicle. By taking torque balance and above assumptions into consideration, the longitudinal motion of vehicle is described by the following equation.

\[ \beta a = \gamma T_a - T_b - T_{es} \]  

where

\[ \beta = \frac{1}{\gamma} \left( J_x + \frac{1}{\pi G} \eta_1 J_y + 4 J_z + m u^2 \right) \]  

\[ \gamma = R R_e \eta_1 \eta_2 \]
Engine & Torque converter: 

\[ T_a = \mu T_e \]  

(4)

where \( \mu \) is the torque gain of the torque converter and \( T_e \) is the engine torque. The characteristics of torque gain \( \mu \) are described by a non-linear function of the ratio between the angular velocity of turbine and pump \( \dot{\xi} = \omega_t / \omega_p \). \( \mu \) can be approximated to 1 at high speed because there is no slip (i.e. \( \dot{\xi} = 1 \)). However, at low speed there is slip between the pump and turbine so \( \dot{\xi} < 1 \) and \( \mu \) takes a maximum value between two and three.

The non-linear behaviour of the engine is described by operating curves that give the ideal static engine torque \( T_e^* \) as a function of throttle angle \( \theta \) and the engine angular velocity \( \omega_e \):

\[ T_e^* = f_{T_e^*}(\omega_e, \theta) \]  

(5)

It has been shown that \( f_{T_e^*}(\omega_e, \theta) \) has the following property [5].

**Property 1:** For any given desired static engine torque \( T_e^* \) and engine angular velocity \( \omega_e \), there exists a unique throttle angle \( \theta \) that achieves the desired static ideal engine torque. In other words, for any \( T_e^* \) and \( \omega_e \), equation (5) has a unique solution \( \theta = f_\theta(\omega_e, T_e^*) \).

The engine dynamics is modeled by a first order lag system with time-delay [6].

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**Brake System** [3]: The brake torque \( J_g \) is defined to be proportional to the brake pressure \( p(t) \):

\[ T_b(t) = k_b p(t) \]  

(6)

The dynamics of the braking pressure system is also modeled with a first-order lag system with time-delay.

**Resistance Forces**: Longitudinal motion is influenced by several disturbances summarized by \( T_{ex} = T_w + T_r + T_c \).

The aerodynamic resistance is given by

\[ T_w = c_a (v - v_w)^2 r \]  

(7)

with the aerodynamic drag coefficient \( c_a \) and the wind velocity \( v_w \). The rolling resistance is

\[ T_r = k_r \cdot c_r \]  

(8)

where \( k_r \) is the coefficient of rolling resistance, \( g \) is gravitational acceleration and \( \alpha \) is the road incline angle. The climb torque is given by

\[ T_c = k_c \cdot c_w \sin(\alpha) \]  

(9)

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**IV. Conclusions**

A longitudinal model using in the design of Stop & Go ACC systems has been discussed. This model includes both the dynamics of the engine and of the brake system. They have been modelled using first order lag systems with time delay. The characteristics of the torque converter have been taken into account as has torque gain. The proposed model is being evaluated and the results will be published elsewhere.

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**References**


