A Complex Lyapunov Theory-based Adaptive Algorithm

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Abstract - This paper presents a complex-valued version of the Lyapunov theory-based adaptive filtering algorithm [1]. The resulting algorithm simultaneously updates the real and imaginary parts of the complex coefficients so that the complex error can converge to zero asymptotically. The proposed scheme can be applied to random and deterministic processes because only the desired signal and input signal are required. The design is independent of the stochastic properties of signals and the stability is guaranteed by the Lyapunov Stability Theory. Simulation example is included to demonstrate the performance of the new complex adaptive algorithm.

I. INTRODUCTION

Most available adaptive filters are real-valued and are suitable for signal processing in real-dimensional space. In some applications, however, some signals are complex-valued and processing is done in complex-dimensional space. An example is the channel equalization of communication channels with complex signaling schemes such as quadrature amplitude modulation (QAM). Another application example is the frequency domain adaptive filtering [12] where the signals and filter coefficients are complex. For complex signal processing problems, many existing adaptive algorithms cannot be applied directly. Although for certain applications it is possible to reformulate a complex signal processing problem so that a real-valued adaptive algorithm can be used to solve the problem, it is not always feasible to do so. Furthermore it is preferred to preserve the concise formulation and elegant structure of complex signals. [3]

Some researchers have proposed the complex adaptive algorithms by extending the real-version of the adaptive algorithms such as LMS (Least Mean Square) and RLS (Recursive Least Square) to the complex forms [3]-[5]. For complex-LMS, the mean squared value of the complex error is minimized according to the complex-LMS algorithm [5]. Widrow [5] showed that the weighting is controlled by the complex-LMS adaptation step-size or learning rate. A large step-size will lead to rapid convergence but the filter parameters may then oscillate or become unstable, while low value implies slow convergence. [4] The major advantage of the LMS lies in its computational simplicity but it is highly dependent on the autocorrelation function associated with the input signals and slow convergence.

Real-RLS algorithm is preferable for fast convergence and it also exhibits consistent convergence properties, but it is computationally expensive to implement even with the availability of the fast algorithm and it exhibits unstable performance [6]. Methods of avoiding instability have been proposed in [7]-[10] but the stability problem of the adaptive filters have not been solved if there are some bounded input disturbances. Similar problems are encountered by the complex-RLS.

Authors [1] have noticed the stability problem of real-RLS adaptive filter and proposed Lyapunov theory-based adaptive filtering (LAF). The design adaptive filter is the modification of RLS algorithm using Lyapunov stability theory. The LAF algorithm [1],[2] is independent of the stochastic properties of the signals. Based on the observations and a collection of desired response, the filter coefficients are updated in the Lyapunov sense so that the error between the desired response and the filter output can asymptotically converge to zero. Furthermore, the stability of LAF filter is guaranteed by Lyapunov stability theory. The LAF in [1] is only designed for real-value adaptive filtering and it cannot be applied to complex-value applications directly. Therefore, a complex adaptive Lyapunov algorithm, which is the extension of the LAF algorithm is proposed in this paper.

II. PROBLEM FORMULATION

The typical structure of an adaptive filtering system is illustrated in Figure 1. For real signal processing, \( x(k) \) is the real input signal of the filter, which has been disturbed by the non-linearity of the communication channel and noises, \( y(k) \) is the real output signal of the filter, \( d(k) \), the real desired response, is provided for the output of the filter to follow, \( e(k) \) is the real error between the desired reference signal \( d(k) \) and the output of the filter \( y(k) \).

\[
e(k) = d(k) - y(k)
\]  

(2.1)

The adaptive algorithm in Figure 1 is generally designed to update the filter real coefficients so that the cost function of the error is minimized in the parameter space.

Authors in [1] have proposed a Lyapunov theory-based adaptive filtering (LAF) algorithm. It has been shown in [11] that, unlike many adaptive filtering schemes using gradient search in the parameter space, LAF algorithm uses a Lyapunov function \( V(k) \), which is positive definite, with a unique global minimum in the state space. By properly choosing the parameter update law in the sense that \( \Delta V(k) = V(k) - V(k-1) \) is negative, the output of the adaptive filter can asymptotically converge to the desired reference signal according to Lyapunov stability theory [11]. Therefore, the local minima problem occurred in the gradient search-based adaptive filters is avoided and at the same time, the stability of the error dynamics is guaranteed. However, the LAF in [1] is only designed for real-valued signal adaptive filtering and it cannot be
applied to complex-value applications directly. Hence a
complex-LAF is needed for complex signal processing or
applications.

For the given complex input signal

\[ y(k) = H(k) x(k) \]

where \( y(k) \) is the complex error

rate of complex error

on the Lyapunov stability theory [11]. The convergence
rate of complex error \( e(k) \) depends on \( \kappa \). It is easy to see
that convergence analysis of the complex Lyapunov
theory-based adaptive algorithm is similar to the real
Lyapunov theory-based adaptive algorithm given in [1]
and can be carried out easily.

Remark 3.2: The complex adaptive gain, \( g(k) \) can be
modified as the expression in (3.7) to prevent the
singularities due to zero values of \( \alpha(k) \) and \( X^*(k)X(k) \).

\[
g(k) = \frac{X^*(k)}{X^T(k)X(k) + \lambda_1} \left( 1 - \kappa \frac{|e(k-1)|}{\alpha(k)} \right) \tag{3.7}
\]

where \( \lambda_1, \lambda_2 \) are small complex numbers, for example, \( \lambda_1 = \lambda_2 = 0.001+j0.001 \). These constants can be chosen to have same values. Smaller these values contribute smaller complex error, \( e(k) \).

IV. SIMULATION EXAMPLE

The performance of the proposed complex-LAF adaptive
algorithm is demonstrated for the complex signal adaptive
filtering. In this simulation, the complex desired response
of the adaptive filter is defined as

\[
d(k) = e^{(-0.01i(1+2j))} + (1 + j)
\]

and the filter complex input signal \( x(k) \) which is corrupted by the additive complex noise, is given by

\[
x(k) = d(k) + w(k)(1+j)
\]

where \( w(k)(1+j) \) is a bounded complex random noise that satisfies the following bounded condition.
0 ≤ w(k) ≤ 0.1.

The adaptive filter has the following structure:

\[ y(k) = h_1(0)x(k) + h_2(1)x(k-1) + h_3(2)x(k-2) \]

The adaptive gain is updated according to the expression (3.7). Initially, the parameters \( \lambda_1, \lambda_2 \) and \( \kappa \) in the expression (3.7) are chosen as follow: \( \lambda_1 = \lambda_2 = 0.1 + 0.1j \), and \( \kappa = 0.1 \).

Parameters \( \lambda_1, \lambda_2 \) are very large.
V. CONCLUSION

This paper has provided a new approach in designing a complex adaptive algorithm using the Lyapunov Stability Theory. The design of this complex filter is independent of stochastic properties of the signals. The proposed scheme can be applied to random and deterministic processes because only the desired signal and input signal are required. The stability of this scheme is guaranteed by the Lyapunov stability theory. This scheme possesses distinct advantages of stability, speed of convergence, computational complexity and robustness to additive noise or disturbance over some complex adaptive algorithms. Simulation example has revealed the performance that can be achieved based on the new complex adaptive algorithm.

REFERENCES


