Discrete-Time Model Reduction of Sampled Systems Using an Enhanced Multiresolutional Dynamic Genetic Algorithm

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Abstract- A framework to automatically generate a reduced-order discrete-time model for the sampled system of a continuous plant preceded by a zero-order hold using an enhanced multiresolutional dynamic genetic algorithms (EMDGA) is proposed in this paper. Chromosomes consisting of the denominator and the numerator parameters of the reduced-order model are coded as a vector with floating point type components and searched by the genetic algorithm. Therefore, a stable optimal reduced-order model satisfying the error range specified can be evolutionarily obtained. Because of the use of the multiresolutional dynamic adaptation algorithm and genetic operators, the convergence rate of the evolution process to search for an optimal reduced-order model can be expedited. Another advantage of this approach is that the reduced discrete-time model evolves based on samples directly taken from the continuous plant, instead of the exact discrete-time model, so that computation time is saved.

1 Introduction

When a continuous plant is subject to digital control [1-3], an equivalent discrete-time system for the sampled system of the continuous plant preceded by a zero-order hold (ZOH) is required, either for digital simulation or design. It is also well known that the exact discrete-time model for the sampled system has the same order as that of its continuous counterpart [1]. As a result, if the original continuous system is of high order, it generally leads to a much greater amount of effort in designing a digital controller for the sampled system, especially in terms of necessary computation.

Recently, genetic algorithms [4-5] have drawn significant attentions in various fields due to their capabilities in directed random search for global optimization. Thanks to a probabilistic search procedure based on the mechanics of natural selection and natural genetics, the genetic algorithms are highly effective and robust over a broad spectrum of problems. This motivates the use of the genetic algorithms to overcome the problem encountered by the conventional optimization methods. Thus, a framework to automatically generate a reduced discrete-time model with a desired order for the sampled system of a continuous plant preceded by a ZOH using genetic algorithms (GA) is proposed in this paper.

2 Discrete-time model reduction of the sampled system via genetic algorithms

It is our intention to automatically generate a stable optimal reduced-order discrete-time model for the sampled system directly from the continuous-time plant \( G_p(s) \), instead of the exact discrete-time model \( G_p(z) \).

Fig. 1 shows the proposed framework of the GA-based model reduction method, in which the enhanced multiresolutional dynamic genetic algorithm (EMDGA) is used to search for the optimal solution available for the denominator and numerator coefficients of the reduced-order model \( G_r(z) \) through an evolution process.

3 Computational Algorithms

The proposed GAs-based model reduction method is supplemented by a computational algorithm below, which can be easily implemented using Matlab.

Step 1: Preparation
Step 2: Initialization
Step 3: Feasibility Check
Step 4: Evaluation
Step 5: Reproduction
Step 7: Combined Crossover
Step 8: Dynamic Mutation
Step 9: Adaptation
Step 10 If \( t = \text{max}_\text{gen} \) or no significant improvement in the best fitness value, then output best solution \( X^* \) with a best fitness value of \( f_{\text{max}} \); else \( t = t + 1 \) and goto Step 3.

4 Illustrated Example

Consider the high-order continuous-time plant [6]
\[
G(s) = \frac{1441.5s^3 + 78319s^2 + 525286125s + 60769325}{s^7 + 1120.4s^6 + 375592s^5 + 3975673s^4 + 36365056s^3 + 759894195s^2 + 68365625s + 617497375}
\]

Fig. 1  Framework of the proposed GA-based model reduction method.
The 3rd-order reduced discrete-time model to be identified for the continuous-time plant preceded by a ZOH using the proposed GA-based approach is

\[ G_r(z) = \frac{b_1z^2 + b_2z + b_3}{a_3z^3 + a_2z^2 + a_1z + a_0} \]

where \(a_0-a_3, b_1-b_3\) are parameters to be identified so that \(G_r(z)\) has a system response close to the exact discrete-time model \(G_{ph}(z)\). Though we do not need \(G_{ph}(z)\) during the identification process by using the proposed approach. For comparison purpose, however, the exact discrete-time transfer function for the continuous-time plant preceded by a ZOH at sampling period \(T=0.05\) Sec. is listed below for reference.

\[ \begin{align*}
\frac{G_{ph}(z)}{G_{numz}} &= \\
&= \frac{(0.00022731z^2 + 0.0011976z^3 - 0.0016836z^4 - 0.00027901z^5 + 0.000061495z^2 - 2.7074 \times 10^{-7}z - 2.7436 \times 10^{-9})}{(z^2 - 4.4978z^3 + 8.3405z^4 - 8.1122z^5 + 4.3021z^6 - 1.1428z^7 + 0.11406z - 0.0036905)}
\end{align*} \]

By using the proposed GA-based approach, an optimal reduced-order model

\[ G(z) = \frac{\text{num}}{\text{den}} \]

is obtained after the 100th generations with the best recorded RMS error = 0.01625378473283 shown in Fig. 2.

In this paper, we have proposed a genetic algorithms-based approach to automatically generate a reduced-order discrete-time model for the sampled system of a continuous plant having a higher order preceded by a zero-order hold. Compared with conventional model reduction techniques, which either lead to no guaranteed performance or encounter nonlinear optimization problems, the GA-based approach provides a reduced model with a closer system response to its original system model in terms of RMS error. Thanks to the multiresolutional dynamic adaptation algorithm and genetic operators, the convergence rate of the evolution process to search for an optimal reduced-order model can be expedited. Another advantage of this approach is that the reduced discrete-time model evolves based on samples directly taken from the continuous plant, instead of the exact discrete-time model, so that computation time is saved. Moreover, cost function to be minimized can be the weighted RMS error of the impulse response, step response, or both, between the reduced-order model and the original system, so that system performance for the reduced-order model can be further improved. Although the problem is multi-peak in nature where solutions might be trapped in local minimum via the GAs approach, this doesn’t prevent the proposed GA-based approach from solving real-world problems of model reduction.

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Bibliography