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# RECONSTRUCTION OF MINIMUM-PHASE SIGNAL WITH MISSING DATA POINTS

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**Abstract:** In many practical situations, the data available for analysis is incomplete due to missing data points. These situations arise either due to malfunctioning of hardware or due to the presence of excessive noise at some of the data points. In such situations, it is desirable to reconstruct the original signal from the incomplete signal. In the present paper, the problem of interpolation of the incomplete minimum-phase signal is considered and an iterative algorithm is proposed for this purpose. The iterative algorithm is based on the property of the minimum-phase signal that its complex cepstrum is a causal real sequence and, hence, can be computed from its even part. The algorithm is applied to a number of signals and results are discussed.

## 1. Introduction

In many practical situations, the data available for analysis is incomplete due to missing data points. These situations arise either due to malfunctioning of hardware or due to the presence of exessive noise at some of the data points where these noisy data points are deliberately discarded. In such situations, it is desirable to reconstruct the original signal from the incomplete signal.

Two types of situations arise in practice. In the first type of practical situations, a block of data is missing due to malfunctioning of hardware or presence of excessive noise. Reconstruction of the signals in these situations has been termed in the literature as the 'signal extrapolation' problem and a number of methods have been reported to solve this problem [1-5,15].

In the second type of practical situations, locations of missing data points in a finite duration data record are random. This means that data available for processing can be considered as unevenly sampled signal. Reconstruction of the signals in these situations has been referred in the literature as the 'signal interpolation' problem and a number of studies have been reported recently in the literature on this problem [6-9]. The Gerschberg-Papoulis algorithm which has been originally proposed for the signal extrapolation problem [1-5] has been applied to the signal interpolation problem for the band-limited signals by a number of authors [6-9]. Nutall [10] has used linear prediction techniques to compute the autoregressive (AR) parameters from the incomplete AR signal and has accomplished signal interpolation using these parameters.

In the present paper, we are interested in the signal interpolation problem. Signal interpolation requires the use of some a priori information about the signal. For example, the linear prediction method of Nutall [10] uses the a priori information that the signal is generated as the output of the AR system. The Gerschberg-Papoulis algorithm [6-9] uses the a priori information that the signal is band-limited spectrally. Interpolation efficiency of an algorithm depends on the a priori information it uses.

In the present paper, we propose an algorithm which assumes the signal to be of minimum phase. Like the Gerschberg-Papoulis algorithm, the present algorithm is also iterative in nature. It is based on the property of the minimum-phase signal that its complex cepstrum is a causal real sequence and, hence, can be computed from its even part [11].

The paper is organized as follows. Section 2 describes briefly some relevant properties of the minimum-phase signal. The minimum-phase signal interpolation algorithm is described in Section 3. Results are presented in Section 4 and Section 5 describes conclusions.

## 2. Some properties of the minimum-phase signal

In this section, we describe some of the relevant proprties of the minimum-phase signals. More detailed description about the minimum-phased signals may be found elsewhere [11-13].

Let $\{v(n)\}$ be a real-valued signal and $\{\hat{v}(n)\}$ be its complex cepstrum. (Note that the complex cepstrum of a real-valued signal is a real-valued sequence.) The signal and its complex cepstrum are related through their z-transforms as follows:

$$\hat{V}(z) = \log V(z).$$

The even- and odd-parts of the complex cepstrum can be computed by taking the inverse discrete Fourier transform (IDFT) of the log-magnitude and phase of V(z) as follows:

$$\hat{v}_e(n) = \mathrm{IDFT}[\log|V(z)|],$$

and

$$\hat{v}_o(n) = \mathrm{IDFT}[j\mathrm{Arg}(V(z))].$$

If the signal $\{v(n)\}$ is a minimum-phase signal, then it has been shown [10] that its complex cepstrum is a causal sequence. This property enables the complex cepstrum to be computed from its even-part as follows:

$$\hat{v}(n) = \begin{cases} \hat{v}_e(n), & \text{for } n=0, \\ 2\hat{v}_e(n), & \text{for } n>0, \\ 0, & \text{for } n<0. \end{cases}$$

These equations can be used to reconstruct the minimum-phase signal when only the magnitude spectrum of the signal is available.

Because of the causal-complex-cepstrum property of the minimum-phase signal, the odd-part of the complex cepstrum can be computed from the even-part as follows:

$$\hat{v}_o(n) = \begin{cases} 0, & \text{for } n=0, \\ \hat{v}_e(n), & \text{for } n>0, \\ -\hat{v}_e(n), & \text{for } n<0. \end{cases}$$

The discrete Fourier transform (DFT) of the odd-part of the complex cepstrum can be used to obtain the unwrapped phase spectrum of the minimum-phase signal [14].

## 3. Minimum-phase signal interpolation algorithm

In this section, we describe the signal interpolation algorithm for reconstructing the minimum-phase signal from the incomplete signal with missing data points.

Let $\{x(n), n=0,1,\ldots,N-1\}$ be the original minimum-phase signal. The (observed) incomplete signal with missing data points can be written as

$$y(n) = x(n)d(n), \quad n=0,1,\ldots,N-1,$$

where $\{d(n)\}$ is the random sequence of 0's and 1's.

The aim here is to reconstruct the minimum-phase signal from the observed incomplete signal $\{y(n)\}$ such that the reconstructed signal is as close to the original signal $\{x(n)\}$ as possible. This is achieved here by using the following iterative algorithm:

(1) Initialize

$$s_0(n) = y(n), \quad n=0,1,\ldots,N-1,$$

and set $i=1$.

(2) Compute the discrete fourier transform (DFT) of the signal $\{s_{i-1}(n)\}$:

$$S(e^{jw}) = \mathrm{DFT}\{s_{i-1}(n)\}.$$

(3) Compute the even part of the complex cepstrum as follows:

$$\{c_e(n)\} = \mathrm{IDFT}[\log|S(e^{jw})|]$$

(4) For the minimum-phase signal, the complex cepstrum is a causal sequence. Therefore, it can be obtained from its even part as follows:

$$c(n) = \begin{cases} c_e(n), & n=\emptyset, N/2, \\ 2c_e(n), & \emptyset<n<N/2, \\ \emptyset, & N/2<n<N-1. \end{cases}$$

(5) Compute the minimum-phase signal $\{\hat{s}(n)\}$ from its complex cepstrum as follows:

$$\{\hat{s}(n)\} = \mathrm{IDFT}[\exp[\mathrm{DFT}\{c(n)\}]]$$

(6) Compute the interpolated signal as follows:

$$s_i(n) = (1-d(n))\hat{s}(n)+y(n), \quad n=\emptyset,1,\ldots,N-1.$$

(7) If the difference,

$$D = \sum_{n=0}^{N-1} [s_i(n)-s_{i-1}(n)]^2,$$

is less than a certain threshold, then stop the iteration process. Otherwise replace i by i+1 and go to step (2).

## 4. Simulation results

We have tried out the present signal interpolation algorithm on a number of minimum-phase signals to restore their missing data points. The results are found to be encouraging on some signals.

For illustration, we consider here a minimum-phase signal with N=7 data points. This signal corresponds to a moving-average (MA) signal. The 7 data points of the signal are: 1.Ø, -Ø.6Ø8Ø, 1.5387, -Ø.3556, 1.4772, -Ø.5454, and Ø.8681. We consider here the case when 3Ø% of the data points are missing.

The iterative algorithm is applied on this example and the reconstruction accuracy is measured in terms of the root-mean-square error between the reconstructed sequence $\{s_i(n)\}$ and the original sequence $\{x(n)\}$. The root-mean-square error is studied as a function of number of iterations for different values of DFT length M (>N). This is shown in Fig. 1. We see from this figure that the root-mean-square error decreases with successive iterations and convergence is reached within 8 to 1Ø iterations. Also, the root-mean-square error is quite high

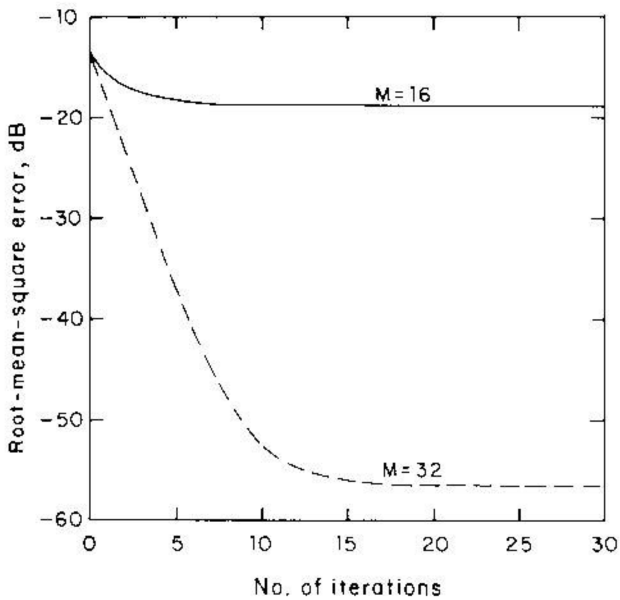


Fig. 1. Root-mean-square reconstruction error as a function of number of iterations (showing the convergence behaviour of the algorithm).

for M=16. This happens due to aliasing in the cepstrum domain. When the DFT length M is increased to 32, the root-mean-square error reduces significantly due to reduction in aliasing effects with increase in M.

We have also tried the signal interpolation algorithm for restoring the missing data points of the minimum-phase AR signals. However, results for AR signals are not found to be encouraging. As mentioned earlier, the interpolation efficiency of a given algorithm depends on the type of a priori information used by the algorithm. The present algorithm uses only the minimum-phase property of the signal. Since the present algorithm does not work satisfactorily for the minimum-phase AR signals, it means that the minimum-phase information does not provide strong enough constraint for interpolation. The interpolation efficiency of the present algorithm can be improved further if other a priori information (such as AR modeling property or band-limited property) can be incorporated along with the minimum-phase property. This will be studied in future.

## 5. Conclusion

In this paper, an interpolation algorithm for the reconstruction of minimum-phase signal from the incomplete signal with missing data points is proposed. The algorithm is iterative in nature and uses the minimum-phase property of the signal. This property states that the complex cepstrum of the minimum-phase signal is a real causal sequence. This allows the computation of complex cepstrum from its even-part which can be obtained from the magnitude spectrum of the signal.

For the minimum-phase MA signals, the iterative algorithm is shown to converge in a relatively small number of iterations. The root-mean-square error in restoring the missing data points of the signal is also found to be quite small. However, the performance of the algorithm is not found to be satisfactory for the minimum-phase AR signals.

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