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A CONSTRAINED FORWARD-BACKWARD CORRELATION PREDICTION METHOD FOR AR SPECTRAL ESTIMATION OF NOISY SIGNALS

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In this paper, a constrained forward-backward correlation prediction (CFBCP) method is proposed for autoregressive (AR) spectral estimation of the noisy signals. This method uses high-order Yule-Walker equations and computes the AR parameters by minimizing the total-squared forward and backward errors in predicting the autocorrelation coefficients under the constraint of the Levinson's recursion relations. Because of this, the CFBCP method has the advantage that it always guarantees the stability of the estimated AR system, even for fixed-point computations. This method is studied for the noisy AR signals as well as for noisy speech and its performance is found to be better than the other methods reported in the literature.

1. INTRODUCTION

For signals following the autoregressive (AR) model, the conventional AR spectral estimation methods (such as the forward-backward linear prediction method and the Burg method) perform reasonably well [1]. But, when these signals are corrupted by the addition of white noise, the AR model is no more valid and, as a consequence, the performance of the conventional methods is poor for noisy signals [2].

It is well known that in the autocorrelation domain only the zeroth order autocorrelation coefficient is affected by the additive white noise, while the other autocorrelation coefficients remain unchanged [2]. Thus, if the use of those Yule-Walker equations which contain zeroth order autocorrelation coefficient can be avoided while computing the AR coefficients, the spectral estimation problem for noisy signals can be solved. This fact has been exploited by many authors [3-5] to develop AR spectral estimation methods for noisy AR signals. In these methods, the high-order Yule-Walker equations are used for estimating the parameters of the pth order AR model. Gersch [3] and Chan and Langford [4] have used an exact number of p high-order Yule-Walker equations to compute p AR parameters. Recently, Cadzow [5] has proposed the use of an overdetermined set of q (>p) high-order Yule-Walker equations for this purpose. In Cadzow's formulation, each high-order autocorrelation coefficient is predicted in terms of p preceding autocorrelation coefficients and the AR parameters are obtained by minimizing the total-squared correlation prediction error. We shall refer this method as the forward correlation prediction (FCP) method. Cadzow has also proposed another method where he has used forward as well as backward prediction of autocorrelation coefficients for estimating the AR parameters. We shall refer this method of Cadzow as the forward-backward correlation prediction (FBCP) method.

A major problem with the FCP and FBCP methods is that these methods do not ensure the stability of the estimated AR system. In many applications (such as speech analysis-synthesis), this stability of the estimated AR system is an important prerequirement [6].

In the present paper, we propose a new method for AR spectral estimation of noisy signals. This method guarantees the stability of the estimated AR system and, at the same time, results in better performance than the FCP and FBCP methods. This method, like the FBCP method, uses forward as well as backward prediction of autocorrelation coefficients. But, to ensure the stability, it minimizes the total-squared forward-backward correlation prediction error under the constraint of Levinson's recursion relations. We shall refer this method as the constrained forward-backward correlation prediction (CFBCP) method.

It might be interesting to note the similarity between the CFBCP method and the conventional Burg method. The only difference between the two methods is that the Burg method works on the time-domain data points, while the CFBCP method works on the autocorrelation coefficients. Because of this, We shall not give here a detailed description of the CFBCP method. Instead, we refer to Anderson's paper [7] for the Burg method and suggest the readers to replace time-domain data points by the autocorrelation coefficients for the CFBCP method. An unbiased estimator is used in the present study for computing the autocorrelation coefficients from the data points.

The organization of this paper is as follows. In Section 2, we present the results of the CFBCP method for the AR signals (i.e., the signals obtained by exciting the AR system with the white Gaussian noise) and compare its performance with that of the Burg, FCP and FBCP methods. Section 3 describes the results for the noisy speech signals. Conclusions are presented in Section 4.

2. RESULTS FOR THE NOISY AR SIGNALS

In this section, we study the CFBCP method for the noisy AR signals. In order to put this method in a proper perspective, we compare here its performance with that of the conventional Burg method and the FCP and the FBCP methods.

For illustration, we select here an example of the AR process which has been commonly used in the literature [2,4]. In this example, the signal is generated from a fourth-order AR system (with parameters $a_1=-2.7607$, $a_2=3.8106$, $a_3=-2.6535$ and $a_4=0.9238$), driven by zero-mean unit-variance white Gaussian noise process. Here the AR signal consists of two narrow-band peaks in its spectrum.

Table I. Statistical performance of different methods in estimating the AR parameters (a_1,a_2,a_3,a_4) for the noisy AR signal at SNR=20 dB with N=40 data points.

	a_1	a_2	a_3	a_4
True values	-2.7607	3.8106	-2.6535	0.9238
Bias-Burg	1.6354	-3.2928	3.0080	-0.9990
Bias-FCP	0.4220	-0.9447	0.8966	-0.3446
Bias-FBCP	0.2631	-0.5937	0.5796	-0.2243
Bias-CFBCP	0.3215	-0.5593	0.4770	-0.1166
Variance-Burg	0.0283	0.0517	0.0396	0.0263
Variance-FCP	0.2327	1.1011	1.0072	0.1555
Variance-FBCP	0.1370	0.7028	0.6700	0.1041
Variance-CFBCP	0.0798	0.3381	0.3197	0.0505
RMS error-Burg	1.6441	3.3007	3.0145	1.0121
RMS error-FCP	0.6409	1.4119	1.3457	0.5237
RMS error-FBCP	0.4541	1.0273	1.0030	0.3929
RMS error-CFBCP	0.4324	0.8067	0.7398	0.2533

We generate 100 independent realizations of the fourth-order AR process and use N=40 data points during steady-state to compute AR parameters for each of these realizations. Different spectral estimation methods are used here with p=4 and p+q=N/2. Statistical bias, variance and root-mean-square (RMS) error in estimating AR parameters are computed by ensemble-averaging over these 100 realizations. Results are listed in Table I for the Burg, the FCP, the FBCP and the CFBCP methods. It can be seen from this table that the statistical performance of the CFBCP method is better than that of the other three methods.

Following Chan and Langford [4], we also define here the resolving capability of the spectral estimation method as the percentage of realizations for which the two peaks are resolved successfully in the spectrum. We have computed the resolving capabilities of the Burg, FCP, FBCP and CFBCP methods which are 4, 43, 42 and 71 percent, respectively. Thus, the CFBCP method resolves the two peaks more often than the other three methods.

3. RESULTS FOR NOISY SPEECH

In this section, we study the CFBCP method for the noisy speech signals and compare its performance with the Burg, the FCP and the FBCP methods. Since the FCP and the FBCP methods result in performance comparable to the CFBCP method, we present here the results for the CFBCP and the Burg methods only.

For illustration, we select here three segments corresponding to vowels /a/, /i/ and /u/ excised from the continuous speech signal digitized at 8 kHz sampling rate. We analyze these segments with N=200, p=10, p+q=N/2 and SNR=10 dB. Power spectral estimates resulting from the Burg and the CFBCP methods are shown in Figs. 1, 2 and 3 for the vowels /a/, /i/ and /u/, respectively. It can be seen from these figures that due to the presence of additive white noise, the higher-order formants in the Burg spectral estimates either get broadened or totally disappear. The CFBCP method restores these higher-order formants and results, in general, better spectral estimates than the Burg method.

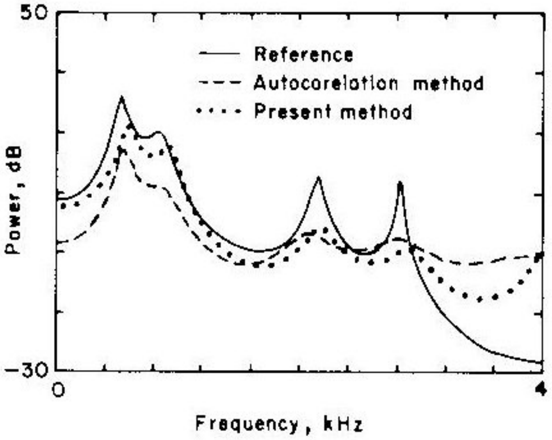

Fig. 1. Estimated power spectra of vowel /a/. (N=200, p=10, q=90 and SNR=10 dB.)

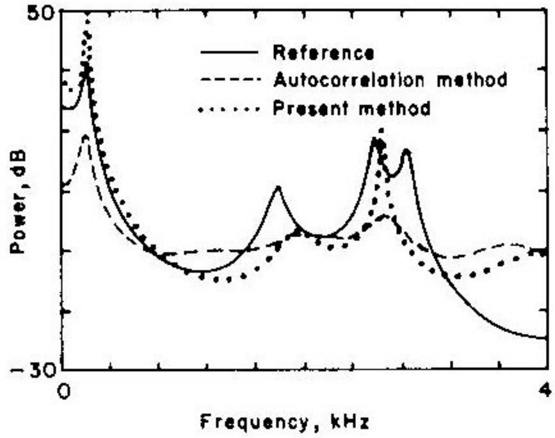

Fig. 2. Estimated power spectra of vowel /i/. (N=200, p=10, q=90 and SNR=10 dB.)

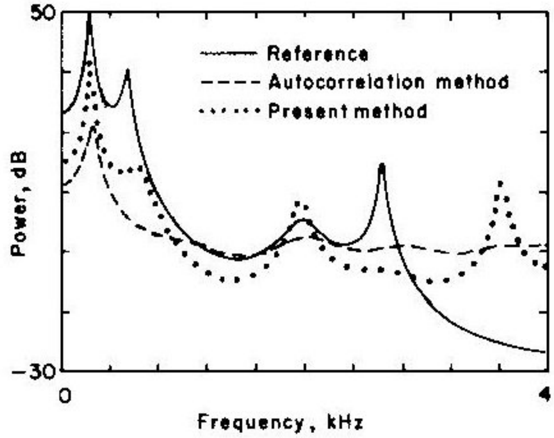

Fig. 3. Estimated power spectra of vowel /u/. (N=200, p=10, q=90 and SNR=10 dB.)

As mentioned earlier, since the CFBCP method (like the Burg method and unlike the other high-order Yule-Walker methods) always guarantees the stability of the estimated AR system, it is specially suited for speech analysis-synthesis applications. We have recently used this method in a multi-pulse excited linear prediction coding system and found that this method results in better quality speecn than the Burg method. These results are reported elsewhere [8,9].

4. CONCLUSIONS

A CFBCP method has been proposed in this paper for AR spectral estimation of the noisy signals. This method has the advantage that it always guarantees the stability of the estimated AR system (even for fixed-point computations) and, thus, it is specially suitable for speech analysis-synthesis applications. We have studied here this method for the noisy AR signals as well as for noisy speech and found its performance to be better than that of the Burg, the FCP and the CFBCP methods.

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