CELL-CONDITIONED MULTISTAGE VECTOR QUANTIZATION

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ABSTRACT — Performance of conventional vector quantizers improves as the rate and/or dimension are increased. However, such improved performance comes at the cost of exponentially increasing complexity. Because of this, use of the conventional vector quantizers is generally limited to lower rates and/or dimensions. One way to reduce this exponential growth in complexity is to use multistage vector quantizers. However, these quantizers are sub-optimal in performance as compared to the conventional single-stage (unstructured) vector quantizers. In this paper, we propose cell-conditioned multistage vector quantizers which reduce the complexity without sacrificing the performance. We show theoretically that under asymptotic conditions the cell-conditioned multistage vector quantizers perform as well as the single-stage vector quantizers. These results are experimentally verified for the vector quantization of speech waveform.

1. INTRODUCTION

Vector quantization (VQ) is a powerful tool for data compression. It has been used in the past [1, 2] in a number of applications (such as speech coding, image coding, etc.). The average distortion of conventional unstructured VQ decreases as the rate and/or dimension are increased. However, such improved performance comes with exponentially increasing complexity. Thus, conventional single-stage VQ is generally limited to lower rates and/or dimensions.

In the literature [1, 2], there are various forms of vector quantizers (VQs) which can reduce the exponential growth in complexity, but at the cost of reduced performance. Most known among these are tree-search, multistage and product-code VQs. In the present paper, we focus our attention to multistage VQ [3] where our intention is to improve its performance, while retaining its advantage of reduced complexity.

For the sake of presentation, we limit here our scope to two stages (as shown in Fig. 1), though the results hold for more than two stages. In two-stage VQ (referred to as 2VQ), after the input vector x is quantized by the first stage, the error $u = x - Q_1(x)$ is quantized by the second stage, and the final reproduction of x is $y = Q_1(x) + Q_2(u)$. If the 2VQ has N_1 quantization points (or, codevectors) in its first stage and N_2 in its second stage, it requires $N_1 + N_2$ memory space and $N_1 + N_2$ distance computations, while the corresponding single-stage VQ would have required $N_1 N_2$ memory space and $N_1 N_2$ distance computations. Thus, with the same rate and dimension, the complexity of two-stage VQ is much less than that of single-stage VQ. However, this reduction in complexity comes at the expense of an increase in distortion.

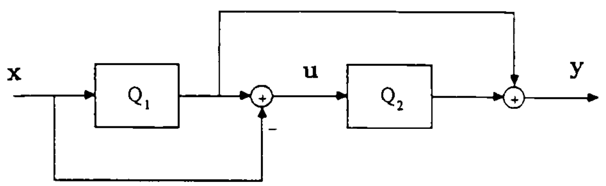

Fig. 1: Two-stage vector quantizer (2VQ).

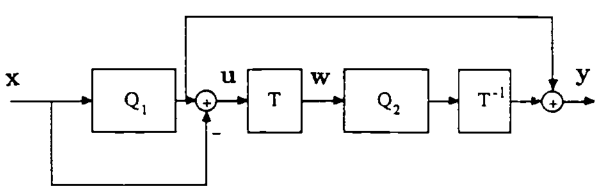

Fig. 2: Cell-conditioned two-stage vector quantizer (CC2VQ).

In order to see why 2VQ gives larger distortion than single-stage VQ with the same dimension and rate, recall the training process where after the first stage, the errors from different cells are pooled together to form the input to the second stage. If all the cells in the first stage have the same size, shape and orientation and if the probability density function (pdf) is flat on each cell, then pooling of errors from different cells is justified. However, this is generally not the case and, as a result, 2VQ is sub-optimal with respect to single-stage VQ.

In order to improve the performance of 2VQ, we propose here a cell-conditioned two-stage vector quantizer (referred to as CC2VQ), which is shown in Fig. 2. Here, the error from the first stage is transformed before and after the second-stage quantization, the transformation being dependent on the first-stage cell to which the input vector X is quantized. The effect of this transformation is, effectively, to cause the size and orientation of different first-stage cells to be as similar as possible. If the number of cells in the first stage is asymptotically large, the pdf within each cell can be assumed to be flat. Moreover, if the first stage is optimally designed, all its cells have approximately the same shape and differ only in their sizes and orientations [4]. Thus a cell-conditioned scaling and rotation allows the pooling of errors from different first-stage cells for quantization by the second stage, without loss of optimality. As a result, performance of CC2VQ will be the same as that of single-stage VQ, at least in the asymptotic case.

In the present paper, we use the mean squared error as the distortion measure. We show theoretically that under asymptotic conditions CC2VQ performs as well as single-stage VQ. In the derivation, we make use of Bennett's integral for VQ distortion as derived by Na and Neuhoff [5] and the Zador-Gersho integral [4]. These results are experimentally verified for VQ of speech waveform.

2. VQ PRELIMINARIES

An N point k-dimensional VQ is characterized by a partition of k-dimensional Euclidean space R^k into N cells $\{S_1, \ldots, S_N\}$ whose centroids define the quantization points $\{y_1, \ldots, y_N\}$. A given k-dimensional source vector x is quantized into one of the y_i's according to the rule $Q(x) = y_i$ if $x \in S_i$. The average distortion incurred by such a quantizer is $D = E \| x - Q(x) \|^2$, where E is an expectation operator and $\| x - y \|$ denotes the Euclidean distance between x and y. The rate of VQ is $R = \log_2 N$ bits per vector (or, $r = (\log_2 N)/k$ bits per sample). Conventional full-search (unstructured) VQ requires storage for N quantization points of dimension k and needs N distance computations.

A key characteristic of a VQ with many quantization points (N large) is its point density, which is a smooth function $\lambda(x)$ whose integral over some region of R^k gives, approximately, the fraction of quantization points in that region. In asymptotic (or high resolution) quantization theory, formulas or bounds for the distortion of a many point quantizer are expressed in terms of point density λ [6, 4, 7, 8, 5]. The most complete such result for VQ is that recently derived by Na and Neuhoff [5] who have shown that under reasonable conditions, the average distortion

$$D \cong \frac{1}{N^{2/k}} \int \frac{m(x)}{\lambda(x)^{2/k}} p_X(x) dx. \quad (1)$$

where $p_X(x)$ is the source density and $m(x)$ is a function, called the inertial profile, that approximately equals the normalized moment of inertia of the quantization cells in the vicinity of x. This distortion is minimized when $\lambda(x)$ is proportional to $p_X(x)^{k/(k-2)}$ and $m(x)$ equals the the constant M_k representing the smallest moment of inertia among all the tesselating polytopes with dimension k [4]. It follows that the minimum average distortion of any k-dimensional quantizer with N quantization points is (for large N)

$$D^* \cong \frac{M_k}{N^{2/k}} \int p_X(x)^{k/(k-2)} dx, \quad (2)$$

which is Gersho's version of Zador's formula [4]

3. CELL-CONDITIONED 2-STAGE VQ (CC2VQ)

In this section, we derive analytically a formula for average distortion for CC2VQ. We show that when the number of quantization points in the first stage is asymptotically large, the average distortion here is the same as that incurred by the conventional single-stage VQ.

In CC2VQ (see Fig. 2), the input vector x is quantized by the first stage and a transformation T is applied to the error vector $u = x - Q_1(x)$ prior to its quantization by the second stage. Thus, the CC2VQ is characterized by this transformation as well as by the attributes of its two stages. Let the first-stage quantizer Q_1 be characterized by N_1 quantization cells $\{S_{1,1}, \ldots, S_{1,N_1}\}$, N_1 quantization points $\{y_{1,1}, \ldots, y_{1,N_1}\}$, point density $\lambda_1(x)$ and inertial profile $m_1(x)$. Similarly, the second-stage quantizer Q_2 is characterized by N_2, $\{S_{2,1}, \ldots, S_{2,N_2}\}$, $\{y_{2,1}, \ldots, y_{2,N_2}\}$, $\lambda_2(x)$ and $m_2(x)$. As mentioned earlier, the effect of transformation T is to cause the size and orientation of different first-stage cells to be as similar as possible[1]. This is accomplished by applying a scaling and rotation to the error vector u, where the amount of scaling and rotation depends on which first-stage cell the input vector x is quantized to. In other words, the transformation T is given by,

$$T = \frac{1}{(V_{1,i})^{1/k}} A_i \qquad \text{when } x \in S_i. \quad (3)$$

where A_i is a $k \times k$ orthogonal matrix representing the rotation, $V_{1,i}$ denotes the volume of $S_{1,i}$ and $1/(V_{1,i})^{1/k}$ is the scaling factor which normalizes the volume of each first-stage cell to unity.

For $x \in S_i$, the governing equations for CC2VQ are

$$u = x - y_{1,i}, \quad (4)$$

$$w = \frac{1}{(V_{1,i})^{1/k}} A_i u, \quad (5)$$

$$y = y_{1,i} + (V_{1,i})^{1/k} A_i^{-1} Q_2(w) \quad (6)$$

We now derive an approximate formula for the average distortion incurred by CC2VQ when the first and second-stage rates are large (i.e., in the asymptotic case). The average distortion can be written as

$$D = \sum_{i=1}^{N_1} D_{1,i} P_{1,i}, \quad (7)$$

where

$$D_{1,i} = \frac{1}{k} E[\| x - y \|^2 \mid x \in S_{1,i}], \quad (8)$$

and

$$P_{1,i} = \int_{S_{1,i}} p_X(x) dx. \quad (9)$$

Assuming N_1 is large, the first-stage cells are small and the source pdf can be considered flat within each of these cells. Under this assumption, we have

$$P_{1,i} \cong p_X(y_{1,i}) V_{1,i}, \quad (10)$$

$$V_{1,i} \cong \frac{1}{N_1 \lambda_1(y_{1,i})} \quad (11)$$

Moreover, there is a widely held conjecture due to Gersho [4] that the cells of the best quantizers are approximately all of the same shape (the tesselating polytope with minimum normalized moment of inertia), though they may differ in size and orientation. If the VQ is designed properly (for example, by using the LBG algorithm [9]), this conjecture can be assumed to be satisfied. From now on, we assume that all the first-stage cells have this optimal shape, which we refer to as the template R. This template has unit volume as can be seen from Eq. (3). It follows that

$$p_W(w \mid x \in S_{1,i}) \cong \begin{cases} 1, & \text{if } w \in R, \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

Using Eqs. (4)-(6), we can write the conditional distortion from Eq. (8) as follows:

$$\begin{aligned} D_{1,i} &= \frac{1}{k}(V_{1,i})^{2/k} E[\| w - Q_2(w) \|^2 \mid x \in S_{1,i}] \\ &\cong (V_{1,i})^{2/k} \frac{1}{N_2^{2/k}} \int_R \frac{M_k}{\lambda_2(w)^{2/k}} dw, \end{aligned} \quad (13)$$

where the last relation uses Eqs. (12) and (1). Substituting Eq. (13) into Eq. (7) and using Eqs. (10)-(11), the average distortion for CC2VQ can be written as

$$\begin{aligned} D &\cong \frac{1}{(N_1 N_2)^{2/k}} \sum_{i=1}^{N_1} \frac{p_X(y_{1,i}) V_{1,i}}{\lambda_1(y_{1,i})^{2/k}} \int_R \frac{M_k}{\lambda_2(w)^{2/k}} dw, \\ &\cong \frac{1}{(N_1 N_2)^{2/k}} \int \frac{p_X(x)}{\lambda_1(x)^{2/k}} dx \int_R \frac{M_k}{\lambda_2(w)^{2/k}} dw, \end{aligned} \quad (14)$$

where in the last relation the summation is replaced by integration (as N_1 is large).

For finding the minimum distortion for CC2VQ, both the integrals in Eq. (14) have to be separately minimized. For minimizing the first integral, the point density $\lambda_1(x)$ should be proportional to $p_X(x)^{k/(k+2)}$ [4]. Also, it can be easily shown that the second integral in Eq. (14) has its minimum when $\lambda_2(w) = 1$ for $w \in R$. This means that the best second-stage quantizer is a uniform quantizer

[1] This choice of transformation is reasonable, as it makes different first-stage cells as similar as possible, under the following assumptions 1) all the first-stage cells are of the same shape, and 2) the source pdf within each of these cells is flat

(which can be implemented perhaps as a lattice quantizer). Aside from the simplicity of uniform quantization, this has the advantage that it need not be redesigned for different sources. With these choices of first- and second-stage quantizers, the minimum average distortion for CC2VQ is

$$D^* \cong \frac{M_k}{(N_1 N_2)^{2/k}} \int p_x(x)^{k/(k+2)} dx. \qquad (15)$$

which is same as the Zador-Gersho formula for the single-stage VQ with $N_1 N_2$ quantization points (see Eq. (2)). Thus, under asymptotic conditions, CC2VQ with N_1 quantization points in the first stage and N_2 quantization points in the second stage performs as well as the single-stage VQ with $N_1 N_2$ quantization points.

4. EXPERIMENTAL RESULTS

In the preceding section, it has been shown theoretically that CC2VQ performs as well as the conventional (unstructured) single-stage VQ under asymptotic conditions. In this section, we try to verify this result experimentally. For this, we study vector quantization of speech waveform and provide results for single-stage VQ, 2VQ and CC2VQ. Results for the first-order Gauss-Markov signal have been reported earlier in [10].

As mentioned earlier, CC2VQ uses a cell-conditioned transformation whose effect is to cause the size and orientations of different first-stage cells to be as similar as possible. This is accomplished by dividing the error vector **u** by a scaling factor and then rotating it (see Eq. (3)). In the present implementation of CC2VQ, we use only scaling (no rotation) on the error vectors. (For large k, the cells of an optimal first-stage VQ are nearly spherical [5] and, hence, rotation may not be necessary.) We show how CC2VQ with only cell-conditioned scaling compares with single-stage VQ and 2VQ.

We use here 120 seconds of speech (from 42 speakers) for training, and 60 seconds of speech (from 20 speakers, different from those used in the training set) for testing. Speech is digitized at 8 kHz. The first-stage VQ of CC2VQ is designed from the training-set speech using the LBG algorithm [9]. In order to compute the scaling factors for different first-stage cells, all the training vectors are encoded. Scaling factor for a given first-stage cell is determined from the training vectors belonging to this cell by using one of the following two methods. The first method uses the square-root of average distortion within the given first-stage cell for computing its scaling factor. In the second method, the square-root of maximum distortion within the given first-stage cell is used to compute its scaling factor. The first method is found to result in better performance than the second method, hence, is used hereafter in this study. Error vectors for all the training vectors are divided by the cell-conditioned scaling factor, and are used for designing the second-stage of CC2VQ using the LBG algorithm. (Note that CC2VQ requires copies of of the first and second-stage codebooks, and the cell-conditioned scaling factors at the transmitter as well as at the receiver.) The single-stage VQ and 2VQ are also designed from the same training-set speech using the LBG algorithm [9, 3]. These VQs are evaluated in terms of their signal-to-noise ratios (SNRs), computed from speech in the test set.

The SNR performance of the single-stage VQ, 2VQ and CC2VQ is studied as a function of bit rate and results are shown in Figs. 3-5 for dimension k equal to 4, 6 and 8, respectively. It can be seen from these figures that CC2VQ performs better than 2VQ for all bit rates and dimensions. But, its performance is not as good as that of the single-stage VQ, though it has been shown theoretically in the preceding section that two VQs are comparable in performance. This

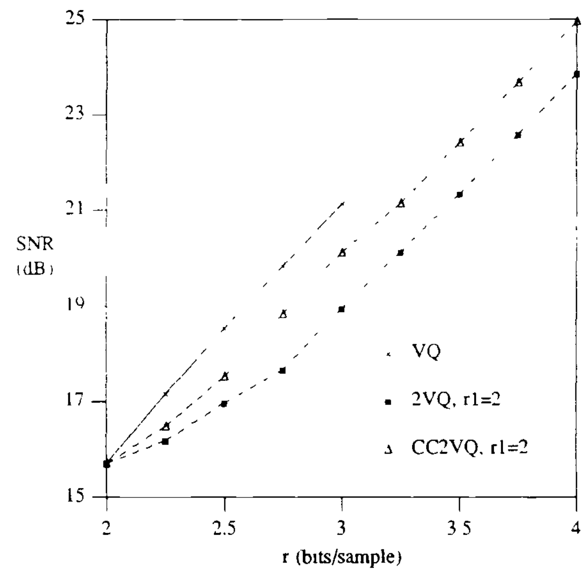

Fig 3: Comparison of single-stage VQ, 2VQ and CC2VQ. $k = 4$.

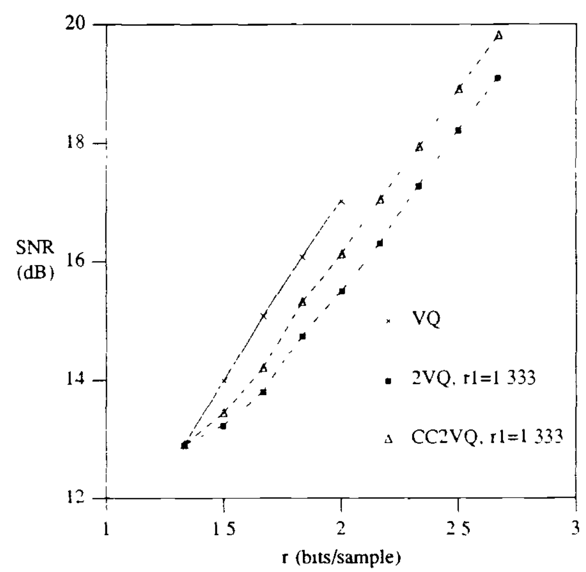

Fig. 4 Comparison of single-stage VQ, 2VQ and CC2VQ. $k = 6$.

can be explained due to the following two reasons. Firstly, CC2VQ, in the present implementation, uses only scaling (and no rotation), while both scaling and rotation are used in the theoretical derivation of distortion for CC2VQ. Secondly, results in Section 3 are derived under asymptotic conditions, specially when N_1 is large so that the source pdf can be assumed flat within each first-stage cell. But, in our experiments, N_1 (or, bit rate for first stage) is not large enough to satisfy this assumption.

In order to see the effect of increasing N_1 on VQ performance, we study SNR performance of 2VQ and CC2VQ as a function

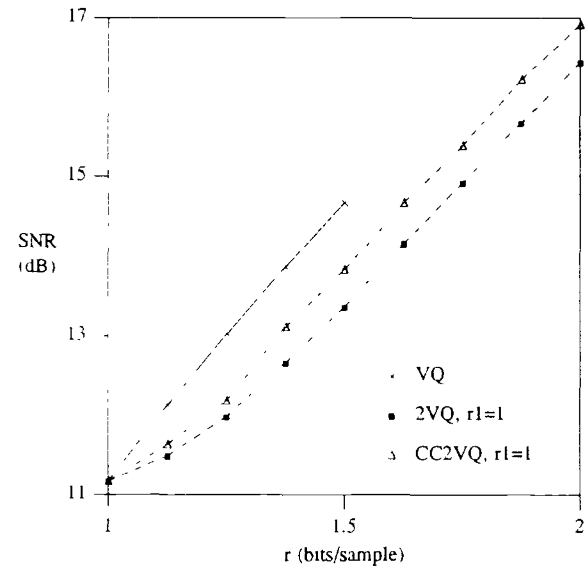

Fig 5 Comparison of single-stage VQ, 2VQ and CC2VQ $k = 8$.

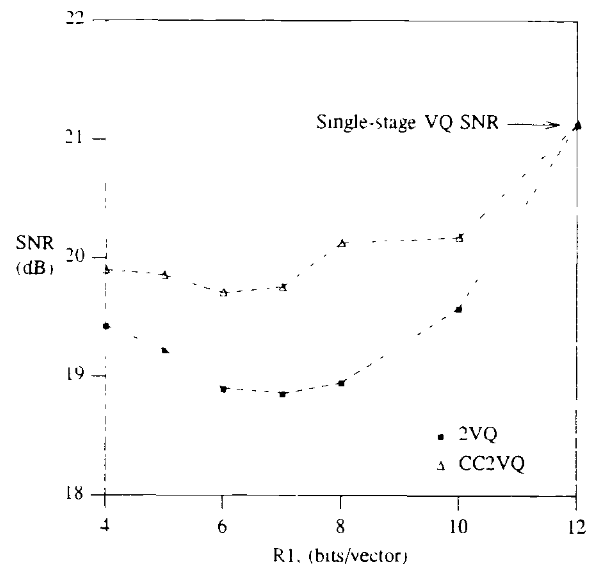

Fig 6 SNR as a function of first-stage bit rate R_1 $k = 4$, $R = R_1 + R_2 = 12$ bits/vector

of first-stage bit rate R_1 $(= \log_2 N_1)$, for the case $k = 4$ and $R = R_1 + R_2 = 12$ bits/vector. (Note that here R_1 is varied, but R $(= R_1 + R_2)$ is kept constant.) Results are shown in Fig. 6. It can be seen from this figure that 2VQ, as expected, initially shows degradation in SNR performance as R_1 is increased, and then its performance starts improving with R_1 — the minimum occurs at R_1 = 6-7 bits/vector. In contrast, CC2VQ's performance does not degrade with R_1, it initially remains approximately constant and then starts improving with R_1. At $R_1 = 12$ bits/vector, both CC2VQ and 2VQ result in the same SNR as obtained by single-stage VQ at $R = 12$ bits/vector. It might be noted here that CC2VQ offers maximum reduction in complexity (in terms of computational cost and memory requirement) at $R_1 = R_2 = R/2$.

5. CONCLUSIONS

As mentioned earlier, 2VQ reduces the complexity of the single-stage VQ, but at the cost of reduced performance. In this paper, CC2VQ has been proposed to improve the performance of 2VQ, while retaining the advantage of reduced complexity. In CC2VQ, the error vector from the first stage is transformed prior to its quantization by second stage. The effect of transformation is to cause size and orientation of different first-stage cells to be as similar as possible. It has been shown theoretically that CC2VQ performs as well as single-stage VQ under asymptotic conditions. Some experimental results for the vector quantization of speech waveform have been presented to support these theoretical results.

References

[1] R.M Gray, "Vector quantization", IEEE ASSP Magazine, pp. 4-29, Apr. 1984

[2] J. Makhoul, S Roucos and H Gish, "Vector quantization in speech coding", Proc. IEEE, vol 73, pp 1551-1588, Nov 1985

[3] B.H. Juang and A.H. Gray, Jr., "Multiple stage vector quantization for speech coding", Proc. ICASSP, pp. 597-600, 1982.

[4] A. Gersho, "Asymptotically optimal block quantization", IEEE Trans. Information Theory, vol. IT-25, pp. 373-380, 1979.

[5] S Na and D.L. Neuhoff, "Bennett's integral for vector quantizers and applications", presented at the IEEE Intern. Symp. on Information Theory, 1990.

[6] W.R. Bennett, "Spectra of quantized signals", Bell Syst. Tech J., vol. 27, pp. 446-472, July 1948.

[7] Y. Yamada, S. Tazaki and R. Gray, "Asymptotic performance of block quantizers with different distortion measures", IEEE Trans. Information Theory, vol. IT-26, pp 6-14, Jan. 1980.

[8] J.A. Bucklew and G.W Wise, "Multidimensional asymptotic quantization theory with r-th power distortion measures", IEEE Trans. Information Theory, vol. IT-28, pp 239-247, Mar 1982.

[9] Y. Linde, A. Buzo and R.M. Gray, "An algorithm for vector quantizer design", IEEE Trans. Commun., vol. COM-28, pp 84-95, Jan. 1980.

[10] D.H. Lee and D.L. Neuhoff, "Conditionally corrected two-stage vector quantization", Proc. 1990 Conf. on Information Sciences and Systems, Princeton, Mar. 1990, pp. 802-806