Frequency errors in AR spectral estimation of sinusoids: A comparative performance evaluation of different modifications over the Burg method

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Abstract: The Burg method of autoregressive (AR) spectral estimation and its three modified versions (recently proposed by Swingler [2], Kaveh and Lippert [3], and Scott and Nikias [4]) are studied as to their effectiveness in estimating the frequency of the sinusoidal signals. The following criteria are used to make a comparative performance evaluation of these four methods: tendency to line splitting, frequency resolution, frequency bias (studied as function of data length, initial phase and input signal-to-noise ratio) and frequency variance. It is found that the modified Burg method of Kaveh and Lippert leads to the best performance among the four methods.

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1. INTRODUCTION

The Burg method of autoregressive (AR) spectral estimation is known to provide high resolution spectral estimates, specially for short data records. Because of this, it is very popular in a number of signal processing applications, such as geophysical data processing, radar, sonar, imaging, speech, radio astronomy, biomedicine, oceanography, etc. However, this method suffers from two major limitaions when applied to short data records of sinusoidal signals [1]. These two limitaions are: relatively high values of bias (or frequency errors) that depend on the initial phase of the sinusoid and the splitting of spectral line under high AR model and large signal-to-noise ratios (SNRs).

Recently, three different modifications over the original Burg method are proposed [2,3,4] which try to reduce the phase-dependence of bias in estimating the sinusoid frequency. These modifications use some form of weighting of the forward and backward prediction error energies prior to their minimization under the constraint of Levinson's recursion. For example, the modification proposed by Swingler [2] uses a Hamming window function for weighting. Kaveh and Lippert [3] have derived a parabolic weighting function by minimizing the average bias in estimating the sinusoid frequency and called this weighting function as the "optimum" tapered window function. In the modification proposed by Scott and Nikias [4], the weight associated with each forward (backward) linear prediction error energy of AR model order m is the measured energy of the m previous (next) data points. In the present paper, we shall refer these weighted Burg methods as the BURGH, BURGO and BURGE methods, respectively.

The aim of the present paper is to compare these four methods (BURG, BURGH, BURGO and BURGE) when used for estimating the frequencies of the sinusoidal signals. These are evaluated in terms of their tendency to line splitting, frequency resolution, frequency bias and frequency variance. Frequency bias is studied as a function of data length, initial phase and input SNR. Since the presence of a "threshold" SNR below which the estimation error increases dramatically is a well known characteristic of a nonlinear estimation problem, we have also determined these threshold SNRs for different methods. Before comparing these methods in the next section, it must be noted here that like the BURG method, the BURGH and BURGO methods ensure the stability of the estimated AR system, while the BURGE method does not do it, Though this does not effect the spectral estimation per se, it is desirable to have this property for some applications (such as speech analysis-synthesis employing AR modelling).

2. SIMULATION RESULTS

In this section, we present the computer simulation results for the different Burg methods when applied to the sinusoidal signals. Comparative performance evaluation of different methods is done in terms of spectral line splitting, frequency resolution, frequency bias and frequency variance. In the following, f is the frequency (in Hz) of the sinusoidal signal, ϕ the initial phase (in degrees), A the amplitude of the sinusoidal signal, f_s the sampling frequency (in Hz), N the number of data samples and M the AR model order used for spectral estimation. In the present paper, we employ a single sinusoid in additive white Gausian noise of variance σ^2 and define $SNR = 10 \log_{10} (A^2/2\sigma^2)$.

2.1. Spectral line splitting

Spectral line splitting is the occurrance of two or more peaks in the estimated spectrum where only one should have been present. This problem was reported for the BURG method by Chen and Stegen [1]. Fougere et al. [5] have documented this problem in detail and reported the worst case of line splitting for odd number of quarter cycles long sinusoidal data with initial phase of 45^o. For illustration, we employ the example ($f=26.25$ Hz, $\phi=45^o$, $A=1$, $f_s=100$ Hz, $N=101$, $M=9$ and SNR=47dB) used by Fougere et al. [5] first and by Scott and Nikias [4] later. Figure 1 shows the power spectra for this example estimated by the four different methods. The BURG method shows the line splitting, while the BURGH and BURGE methods reduce this tendency to line splitting. The BURGO methods shows no line splitting.

2.2. Frequency resolution

In order to illustrate the frequency resolution results for different methods, we employ here the example used by Scott and Nikias [4]; i.e., $f=1$ Hz, $\phi=135^o$, $A=1$, $f_s=20$ Hz and $N=15$. We show here the results for three different noise conditions: 1) noise-free, 2) SNR=37 dB and 3) SNR=17 dB. For the noise-free case, we employ AR model order M equal to 2, following Swingler [2]. Figure 2 shows the estimated power spectra generated for this case by the four methods. It can be seen from this figure that the BURGO method results in the sharpest peak and, thus, exhibits the best resolution among the four methods. For SNR=37 dB and SNR=17 dB, we employ AR model order M=9. The power spectra are shown in Figures 3 and 4 for these two noise conditions. It can be seen from these figures that the BURGO method preserves the best resolution property for lower SNR values.

2.3. Frequency bias as a function of data length

Here the maximum absolute frequency bias, found by varying the initial phase ϕ, is studied as a function of data length N. The same example as used in the earlier sub-section 2.2 is employed; i.e., $f=1$ Hz, $A=1$ and $f_s=20$ Hz. No noise is added to the data. The AR model order M equal to 2 is used for spectrum estimation. A range of data lengths from N=15 to N=45 is investigated. Figure 5 shows the maximum absolute frequency bias as a function of data length N for all the four methods. It can be seen from this figure that the bias reduces as more and more data points are included in the estimation process and the minimum bias occurs at points half-cycle apart (an observation made earlier by Chen and Stegen [1] for the BURG method and by Swingler [2] for the BURGH method). However, we make here one new observation; i.e., the BURGO method shows minimum frequency bias for data lengths which correspond to maximum frequency bias for the other three methods (BURG, BURGH AND BURGE). This is an important property which has allowed us to combine the BURGO and BURGE methods into a new method which will be reported elsewhere [6]. We also observe from Fig. 5 that for short data lengths, the BURGO method performs best among the four methods, but for large data lengths, the BURGH method gives the best performance.

2.4. Frequency bias as a function of initial phase

For studying the frequency bias as a function of initial phase ϕ, we employ the same example as used in sub-sections 2.2 and 2.3; i.e., $f=1$ Hz, $A=1$, $f_s=20$ Hz and $N=15$. It is studied for three different noise conditions: 1) noise-free, 2) SNR=35 dB and 3) SNR=15 dB. For noise-free case, following Swingler [2], M=2 was employed. For SNR=35 dB and SNR=15 dB, M=9 was used. Figures 6 and 7 show the frequency bias as function of initial phase for the noise free case and for SNR=15 dB, respectively. Fifty different realizations of white Gaussian noise are used. The value of phase-averaged absolute bias was computed for each of the three noise conditions and for each of the four different methods. These values are listed in Table 1. It can be seen from Figures 6 and 7 and from Table 1 that the BURGO method results in the lowest bias for all the three noise conditions.

2.5. Frequency bias as a function of input SNR

The frequency bias is studied here as a function of input SNR for the same example as used before; i.e., f=1 Hz, A=1, f_s=20 Hz, N=15 and M=9. A range of input SNRs from SNR=40 dB to SNR=0 dB is investigated. The resulting variation of frequency bias as a function of SNR is shown in Fig. 8 for the initial phase fixed to 0^o and in Fig. 9 for the random initial phase unformly distributed between 0^o and 360^o. 100 different realizations of white Gaussian noise are used here to compute ensemble average. It can be seen from these figures that the frequency bias is lowest for the BURGO method. As mentioned earlier, we observe in these figures the presence of a "threshold" SNR below which the frequency bias increases dramatically. The values of threshold SNRs determined from these figures are 7.5 dB for the BURG and BURGH methods, 5 dB for the BURGO method and 10 dB for the BURGE method. Thus, in terms of threshold SNR, the BURGO method results in the best performance among the four methods.

2.6. Frequency variance

Here we study the variance in estimating the frequency of the sinusoidal signal for the same example as used in the earlier sub-sections; i.e., f=1 Hz, A=1, f_s=20 Hz, N=15, M=9 and SNR=25 dB. Ten different realizations of white Gaussian noise are used. We show in Fig. 10 the power spectra generated for this example by the four different Burg methods with random initial phase uniformly distributed between 0^o and 360^o. It can be seen from this figure that spread in the peak position is lowest for the BURGO method. The variance in estimating the sinusoid frequency is computed for each method and found to be 0.01588 Hz^2 for the BURG method, 0.00818 Hz^2 for the BURGH method, 0.00027 Hz^2 for the BURGO method and 0.00601 Hz^2 for the BURGE method. Thus, we see that the BURGO method leads to the least variance in estimating the frequency of the sinusoidal signal.

Thus, we have seen that for all the criteria employed in the present study (namely, tendency to line splitting, frequency resolution, frequency bias and frequency variance), the BURGO method performs consistently better than the other Burg methods. It might be noted here that we have also conducted a similar comparative study of different Burg methods for pitch-synchronous analysis of voiced speech and found the performance of the BURGO method to be the best there, too. Results of this study are reported elsewhere [7].

3. CONCLUSION

The BURG method and its three modified versions (BURGH, BURGO and BURGE) are studied for estimating the frequency of the sinusoidal signals. A comparative performance evaluation of these methods is done using the following criteria: tendency to line splitting, frequency resolution, frequency bias (as function of data length, initial phase and input SNR) and frequency variance. It is concluded that the BURGO method results in the best performance among the four methods.

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Table 1. Phase-averaged absolute bias (in Hz) for the four Burg methods at three different noise conditions.

Method	Bias for		
	Noise-free	SNR= 35 dB	SNR= 15 dB
BURG	0.140	0.090	0.127
BURGH	0.100	0.110	0.107
BURGO	0.018	0.025	0.033
BURGE	0.095	0.052	0.083

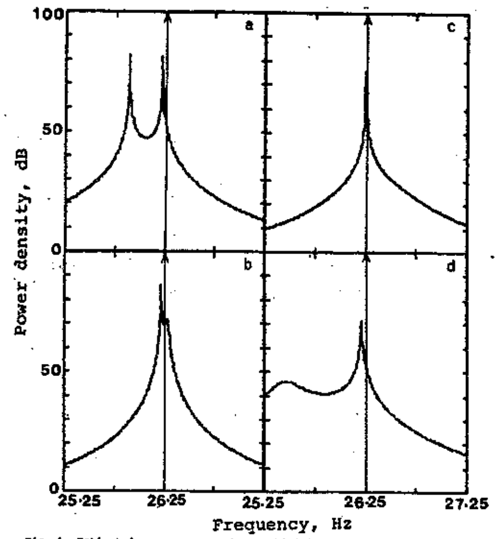

Fig. 1. Estimated power spectra exhibiting spectral line-splitting. Sinusoidal signal (f=26.25 Hz, ϕ=45°, A=1, f_s=100 Hz, N=101, M=9, SNR=47 dB). (a) BURG method, (b) BURGH method, (c) BURGO method, (d) BURGE method.

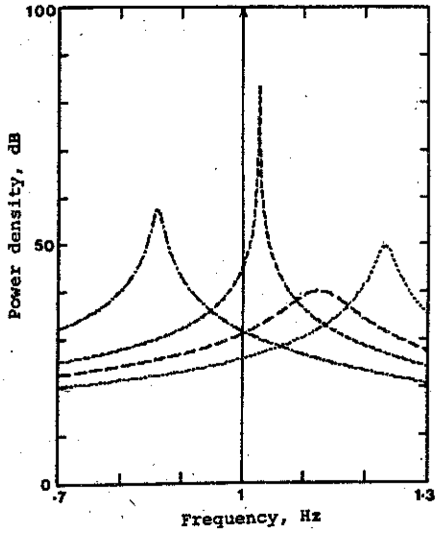

Fig. 2. Estimated power spectra exhibiting frequency resolution. Noise-free sinusoidal signal (f=1 Hz, ϕ=135°, A=1, f_s=20 Hz, N=15, M=2). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

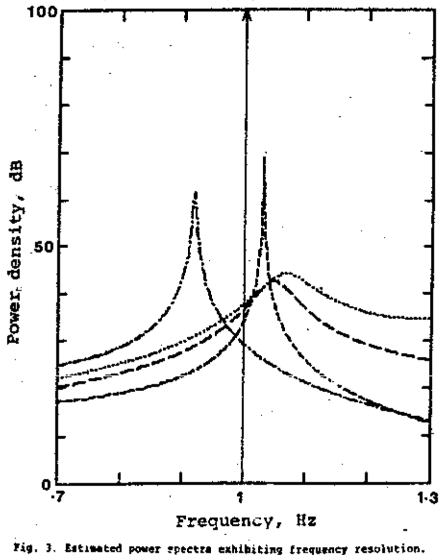

Fig. 3. Estimated power spectra exhibiting frequency resolution. Sinusoidal signal with additive white noise SNR=37 dB (f=1 Hz, ϕ=135° A=1, f_s=20 Hz, N=15, M=9). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

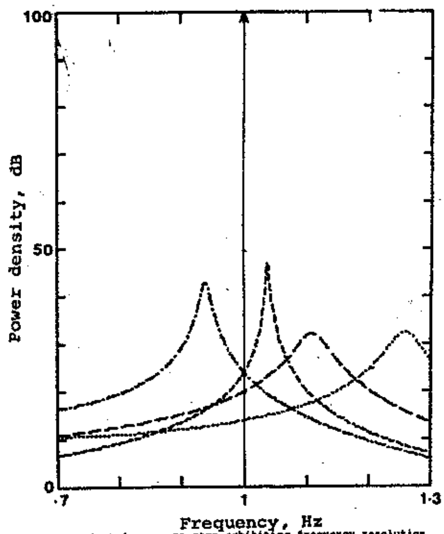

Fig. 4. Estimated power spectra exhibiting frequency resolution. Sinusoidal signal with additive white noise SNR = 17 dB (f=1 Hz, ϕ=135°, A=1, f_s=20 Hz, N=15, M=9). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

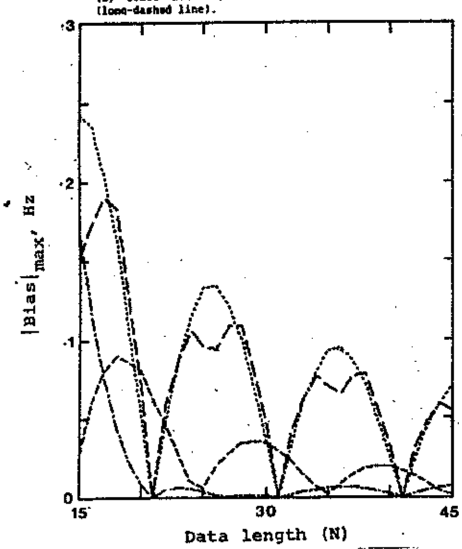

Fig. 5. Maximum absolute frequency bias as a function of data length. Noise-free sinusoidal signal (f=1 Hz, A=1, f_s=20 Hz, M=2). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

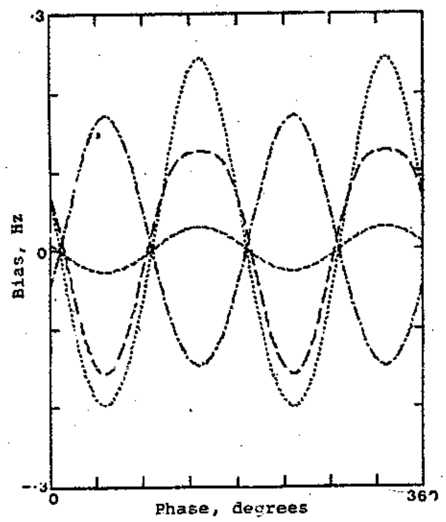

Fig. 6. Frequency bias as a function of initial phase. Noise-free sinusoidal signal (f=1 Hz, A=1, f_s=20 Hz, N=15, M=2). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

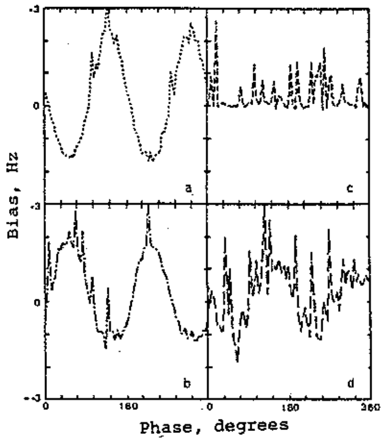

Fig. 7. Frequency bias as a function of initial phase. Sinusoidal signal with additive white noise SNR=15 dB (f=1 Hz, A=1, f_s=20 Hz, N=15, M=9). (a) BURG method, (b) BURGH method, (c) BURGO method, (d) BURGE method.

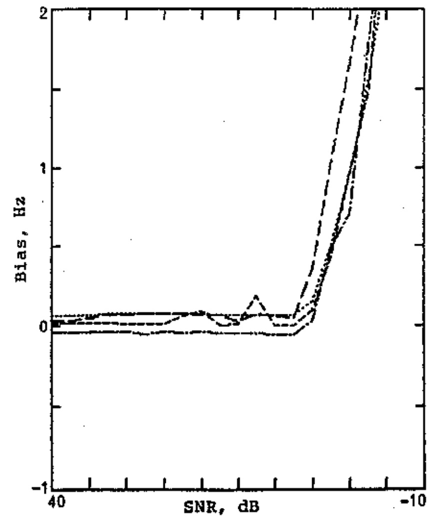

Fig. 8. Frequency bias as a function of SNR. Sinusoidal signal with fixed initial phase of 0° (f=1 Hz, A=1, f_s=20 Hz, N=15, M=9). (a) BURG method (dotted line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

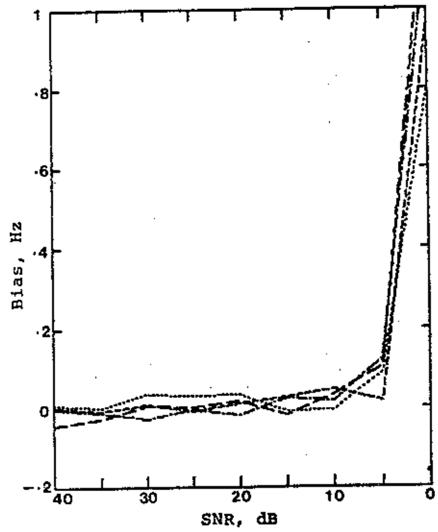

Fig.9. Frequency bias as a function of SNR. Sinusoidal signal with random initial phase uniformly distributed between 0° and 360° (f=1 Hz, A=1, f_s=20 Hz, N=15, M=9). (a) BURG method (dotted-line), (b) BURGH method (dash-dot line), (c) BURGO method (short-dashed line), (d) BURGE method (long-dashed line).

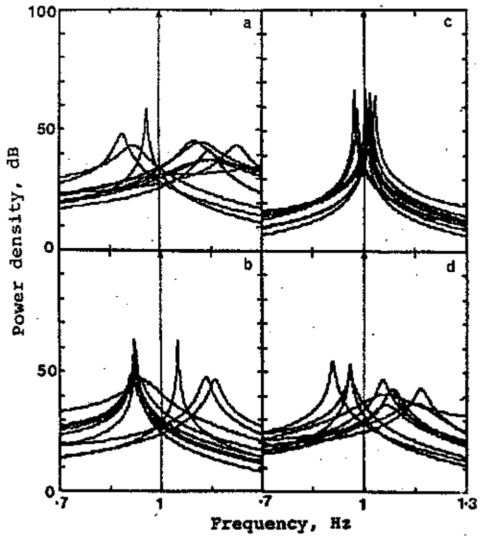

Fig.10. Estimated power spectra exhibiting the variance in estimating sinusoid frequency. Sinusoidal signal with additive white noise SNR=25 dB (f=1 Hz, A=1, f_s=20 Hz, N=15, M=9). (a) BURG method, (b) BURGH method, (c) BURGO method, (d) BURGE method.