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SHORT COMMUNICATION

SOME COMMENTS ABOUT THE ITERATIVE FILTERING ALGORITHM FOR SPECTRAL ESTIMATION OF SINUSOIDS

K.K. PALIWAL

Computer Systems and Communications Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India

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Abstract. Recently, an iterative filtering algorithm has been proposed in the literature for estimating the frequencies of the sinusoids at low signal-to-noise ratios. In this paper, we study the performance of this algorithm for the real sinusoidal signals and suggest the use of a better autoregressive parameter estimation method for filter design. We also make some comments about the convergence behaviour of this algorithm.

Zusammenfassung. Vor kurzem wurde in der Literatur ein iterativer Filteralgorithmus vorgestellt, mit dem die Frequenzen von Sinusfunktionen bei geringem Störabstand geschätzt werden können. Im vorliegenden Beitrag untersuchen wir das Verhalten dieses Algorithmus für reale sinusformige Signale und schlagen vor, beim Entwurf des Filters die autoregressive Schätzmethode für die Filterparameter zu verbessern. Ebenfalls untersucht wird das Konvergenzverhalten dieses Algorithmus.

Résumé. Un algorithme de filtrage itérative a été récemment proposé dans la littérature pour estimer les fréquences des sinusoides à de faibles rapports signal sur bruit. Dans cet article nous étudions les performances de cet algorithme et nous suggérons l'utilisation d'une meilleure méthode d'estimation de paramêtre autoregressive pour élaborer des filtres. Nous faisons aussi des commentaires sur le comportement à la convergence de cet algorithme.

Keywords. Spectral estimation, autoregressive modeling, Burg method, sinusoidal signals, iterative filtering.

1. Introduction

Recently, Kay [4] has proposed an iterative filtering algorithm (IFA) for estimating the frequencies of the sinusoids at low signal-to-noise ratios (SNRs). In this algorithm, the frequency estimation performance is improved by iteratively applying the autoregressive (AR) filter on the noisy data. The AR filter (which has to be stable in this case) can be designed by using any AR spectral estimation method [5]. In his implementation, Kay [4] has used the Burg method for AR parameter estimation and shown that the IFA can provide accurate frequency estimates at very low SNRs. But, at high SNRs, its performance is poor. This is due to the biased nature of the AR parameter estimates resulting from the Burg method. A better AR parameter estimation method which can guarantee the stability of the filter might be able to improve the IFA's performance at higher SNRs.

Recently, some modified Burg methods have been proposed in the literature [2, 8, 9], which result in less biased estimates than the Burg method. In our earlier papers [6, 7], we studied these modified Burg methods and found the optimum-tapered Burg method to lead to the least bias. The aim of the present short communication is to use the optimum-tapered Burg method for

AR parameter estimation and investigate the resulting improvement in the performance of the iterative filtering algorithm for the finite data records of the real sinusoidal signals. In the following, we shall refer to the original Burg method and the optimum-tapered Burg method as the BURG and BURGO methods, respectively.

Kay [4] has shown analytically that, in the asymptotic case (i.e., for infinite data record lengths), the poles of the AR filter move closer to the unit circle with each successive iteration and frequency estimation becomes more accurate, and a convergence is reached after a finite number of iterations. However, for finite data records it may not be true. In fact, during the process of the present investigation we observed that the frequency estimation performance initially improves with each iteration, but after a certain number of iterations it starts deteriorating. This can be explained as follows. Though filtering of input data helps in improving the accuracy of frequency estimates by moving the poles towards the unit circle, it also introduces transient in the beginning of the filtered data when applied with zero initial conditions [4]. The transient duration increases with successive iterations and it is possible that after a certain number of iterations the IFA's performance may start deteriorating. The question that becomes important in this context is when to stop the iteration process. We shall try to answer this question in the present paper.

2. Simulation results

Here, we study, through simulation, the performance of the IFA with BURG and BURGO estimates. To illustrate the results, we employ here an example where the data set consists of two real sinusoids in white noise. In this example, the values of different parameters are taken to be the same as used by Kay [4] in his illustrating example. That is, the frequencies, phases and amplitudes of two sinusoids are: $f_1 = 0.2$ Hz, $f_2 = 0.22$ Hz, $\phi_1 = 0$, $\phi_2 = \frac{1}{4}\pi$, $A_1 = A_2 = 1$. The sampling frequency is 1 Hz. The noise variance σ^2 is adjusted to yield a desired SNR where $\mathrm{SNR} = 10\log_{10}(1/2\sigma^2)$ dB. The data record length is $N = 25$ real points. The IFA uses the correct filter order $p = 4$. The frequencies of the sinusoids are obtained from the estimated power spectrum by the peak-picking method.

First, we want to study the frequency estimation performance of the IFA with BURG and BURGO estimates at different iterations. For this, we use a single realization of signal at SNR = 25 dB. The frequency estimates from the two IFAs are listed in Tables 1 and 2 for different iterations. We can observe from these tables that the frequency estimation performance improves from the first iteration to the second and then it starts deteriorating.

Table 1
Estimates of frequencies f_1 and f_2 and the AR coefficient a_p at different iterations by the IFA with BURG estimates

Iteration	f_1	f_2	a_p
1	0.1970	0.1970	0.7401
2	0.1977	0.2297	0.9845
3	0.1971	0.2298	0.9678
4	0.1854	0.2367	0.8619
5	0.1524	0.2248	0.7445

Table 2
Estimates of frequencies f_1 and f_2 and the AR coefficient a_p at different iterations by the IFA with BURGO estimates

Iteration	f_1	f_2	a_p
1	0.2018	0.2018	0.7050
2	0.2009	0.2238	0.9761
3	0.1937	0.2318	0.9441
4	0.1849	0.2375	0.8462
5	0.1741	0.2354	0.7674

As mentioned earlier, this happens due to the transient introduced due to filtering of input data with zero initial condition. (It might be noted that we have tried to reduce the transient due to filtering by using an algorithm proposed by Kay [3] to set the initial conditions but found the resulting IFA performing inferior to the IFA with zero initial conditions.) At the second iteration where both the IFAs result in best performance, the IFA with BURGO estimates gives more accurate frequency estimates than that with the BURG estimates. Since the IFA's performance deteriorates after a certain number of iterations, it becomes necessary to know when to stop the iteration process. As noted earlier, the IFA tries to improve the frequency estimation performance by moving the poles closer to the unit circle with each iteration. So, the performance is going to be the best at the iteration where the poles are closest to the unit circle. To measure how close the poles are to the unit circle, here we propose to use the last AR coefficient of the filter; i.e., the coefficient a_p of the pth order filter. The coefficient a_p increases as the poles move towards the unit circle and becomes 1 when the poles reach the unit circle. In the last columns of Tables 1 and 2 we list the values of a_p at different iterations. We observe that the IFA's performance is best at the iteration where a_p is maximum. So, we stop the iteration process when a_p starts decreasing.

Next, we study the frequency estimation performance of the two IFAs as a function of SNR. For this, we use 20 different realizations of white Gaussian noise for each SNR and compute the root-mean-square (RMS) error in dB in estimating the sinusoids' frequencies. Fig. 1 shows the RMS error (in dB) in estimating the frequency f_2 as a function of SNR. It can be seen from this figure that the IFA with BURGO estimates results in more accurate f_2 estimation than the IFA with BURG estimates for all SNRs. Similar results are obtained for f_1 estimation.

Following Kay [4], we now compare the performance of the two IFAs with the Cramer-Rao (CR) lower bound and the principal eigenvector (PE) method of Tufts and Kumaresan [10]. For the PE method we use the model order $p = 18$ which gives the best performance [10]. The CR bound and the results from the PE method are shown in Fig. 1. It can be seen from this figure that the 'threshold' SNR (which is defined as the SNR below which the performance deteriorates drastically) is about 7 dB for the PE method, -2 dB for the IFA with

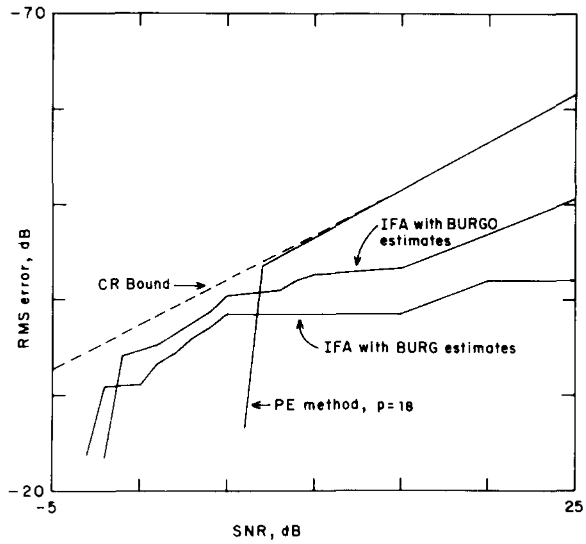

Fig. 1. RMS error in estimating the frequency f_2 of the sinusoidal signal as a function of input SNR.

the BURG estimates and -1 dB with the BURGO estimates. Thus, at lower SNRs, the IFA results in better performance than the PE method (also observed by Kay [4]). But, at higher SNRs (greater than 7 dB) its performance is inferior to the PE method.

We have also studied the performance of these methods for other examples of sinusoidal signals containing one and two sinusoidal components in white Gaussian noise. In terms of frequency estimation errors, we have obtained, for all these examples, results similar to those described earlier in this section.

Thus, we have seen that though the use of BURGO estimates of AR parameters improves the frequency estimation performance of the IFA for higher SNRs, its performance is still not comparable to that of the PE method. However, the present investigation has shown that this inferior performance of the IFA at higher SNRs is due to the biased estimates of AR parameters by the Burg method and a method better than the BURGO method may improve its performance further. Fougere's method [1] is one such method which gives much better estimates of AR parameters than the Burg method and ensures the stability of the filter. We are investigating its use in the IFA and the results will be reported in future work.

It might be noted that the IFA algorithm requires the knowledge of the AR model order [4]. Theoretically, the order of the AR model, p, should be equal to twice the number of sinusoids in the real-valued signal [4]. But, in practice, it can be estimated by using any of the AR model order selection algorithms described in the literature [5]. Kay [4] has studied through simulations the effect of incorrect estimation of p. He has shown that if the estimated value of p is too large, the estimated spectrum shows spurious peaks for short data records and low SNRs. But, for large data records and high SNRs, no spurious peaks have been observed. However, in the present paper, we did not attempt to estimate the AR model order. Instead, we used its theoretical value in the simulations reported here.

3. Conclusions

In the present paper, we have studied the performance of the IFA for finite data records of the real sinusoidal signals and shown that it does not always converge. Initially, its performance improves with successive iterations, but after a certain number of iterations it starts deteriorating. We have proposed a measure which decides when to stop the iteration process. The performance of the IFA is studied with the BURG and BURGO estimates. It is shown that its performance with the BURGO estimates is better than that with the BURG estimates for all SNRs. But, it is still inferior to the PE method at higher SNRs.

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