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REDUCED-DELAY QUADRATURE MIRROR FILTER STRUCTURES FOR SUBBAND CODING OF SPEECH

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Abstract. In subband coding systems of speech, quadrature mirror filter (QMF) banks have been used effectively in a tree-structured form for decomposition and alias-free reconstruction of the speech signal. In the present paper, we derive some new causal and noncausal QMF structures which can reduce group delay. These structures are based on even- as well as odd-length finite impulse response filters.

Zusammenfassung. Quadratur-Spiegelfilterbänke (QMF-Bänke) werden in der Teilband-Sprachkodierung häufig eingesetzt. In einer Baum-Struktur zerlegen sie das Signal und erlauben es auf gleichem Wege ohne Überfaltungsfehler wieder zusammenzusetzen. Im weiteren werden einige neue QMF-Strukturen hergeleitet, die mit nichtrekursiven Filtern gerader oder ungerader Längen arbeiten. Darüber hinaus wird gezeigt, wie einige dieser Strukturen in Teilbandkodern die Laufzeit verringern können, ohne die Qualität des rekonstruierten Sprachsignals zu verringern.

Résumé. Dans le codage en sous bandes de la parole, des bancs de filtres miroirs en quadrature (QMF) ont été utilisés efficacement, dans une structure en arbre, pour la décomposition et la reconstitution parfaite du signal de parole. Dans cet article nous dérivons quelques structure causales et non-causales de QMF qui peuvent réduire le retard de groupe. Ces structures sont basées sur des filtres à réponse impulsionnelle de durée paire et impaire.

Keywords. Speech coding, quadrature mirror filter, group delay.

1. Introduction

The subband coding scheme has been reported to be effective in transmitting speech at medium bit-rates (in the range of 12 to 24 kbits/s) [1, 4]. In this scheme, half-band quadrature mirror filters (QMF's) have been used in the form of a tree-structured bank of filters to split speech into a number of frequency bands at the transmission end and to reconstruct it without aliasing distortion at the receiving end [2]. A QMF structure using the symmetric finite impulse response (FIR) filters of even lengths has been proposed by Esteban and Galand [2] for this purpose.

Recently, Galand and Nussbaumer [3] have introduced a new structure where the QMF's can be of odd lengths. They have shown that these odd-length QMF's can be implemented with a reduced number of multiplications and additions. However, the Galand and Nussbaumer structure has the disadvantage that it allows for the odd-length filters at the cost of increased group delay [3]. (If M is the length of the FIR filter used in a two-band subband coder, the Galand and Nussbaumer QMF structure [3] introduces a group delay of M samples, while the conventional QMF structure [2] introduces a group delay of only $(M-1)$ samples.)

In the present paper, we derive a number of new causal as well as noncausal QMF structures. The noncausal QMF structures can be used only with those subband coders where speech is processed in a block mode. The causal structures, on the other hand, can be used for any type of subband coder; i.e., it does not matter for these structures whether speech is processed in block-wise fashion. We show in the present paper that it is possible to use odd-length QMF's (with causal as well as noncausal structures) without increasing the group delay. We also show that the group delay can be reduced further to $(M-2)$ samples in noncausal structures using even-length QMF's. It might be noted here that this reduction in group delay is possible without affecting the quality of the reconstructed signal.

2. New QMF structures

In order to derive the QMF structures, we consider a two-band subband coding system which is shown in Fig. 1. Here, the input signal $x(n)$ sampled at f_s $(=\frac{1}{2}w_s/\pi)$ is band-split about frequency $\frac{1}{4}f_s$ by the half-band filters $H_1(z)$ and $H_2(z)$. The resulting signals $x_1(n)$ and $x_2(n)$ are down-sampled by the down-samplers D_1 and D_2 at $\frac{1}{2}f_s$ by discarding every second sample. The down-sampled signals are encoded and transmitted. At the receiving end, the signals are decoded and up-sampled by the up-samplers U_1 and U_2 at the original sampling rate f_s by inserting a zero-valued sample after each received sample. The up-sampled signals $\hat{x}_1(n)$ and $\hat{x}_2(n)$ are filtered by $K_1(z)$ and $K_2(z)$ and combined to form the reconstructed signal $\hat{x}(n)$. For the purpose of the present derivation, we assume the transmission channel to be ideal; i.e., $\hat{y}_1(n)=y_1(n)$ and $\hat{y}_2(n)=y_2(n)$.

From Fig. 1, the z-transform $\hat{X}(z)$ of the reconstructed signal $\hat{x}(n)$ can be written in terms of the z-transform $X(z)$ of the input signal $x(n)$ as follows:

$$\hat{X}(z)=\tfrac{1}{2}\{H_1(z)K_1(z)+H_2(z)K_2(z)\}X(z)+\tfrac{1}{2}\{H_1(-z)K_1(z)+H_2(-z)K_2(z)\}X(-z). \quad (1)$$

In this expression, the second term arises due to aliasing effects resulting from the down-sampling and the up-sampling operations. If the half-band filters $H_1(z)$ and $H_2(z)$ are taken to be $H(z)$ and $H(-z)$, respectively, then Esteban and Galand [2] have shown that the aliasing term in (1) can be made zero by selecting filters $K_1(z)$ and $K_2(z)$ as $H(z)$ and $-H(-z)$, respectively. The resulting z-transform $\hat{X}(z)$ of the output signal is then given by

$$\hat{X}(z)=\tfrac{1}{2}[H^2(z)-H^2(-z)]X(z). \quad (2)$$

If $H(z)$ is a symmetrical M-tap FIR filter having low ripple in passband and high rejection in the stopband, then Esteban and Galand [2] have shown that the output signal $\hat{x}(n)$ will be a replica of the input signal $x(n)$ (except for a scale factor of 2 and a group delay of $(M-1)$ samples) provided the filter length M is even.

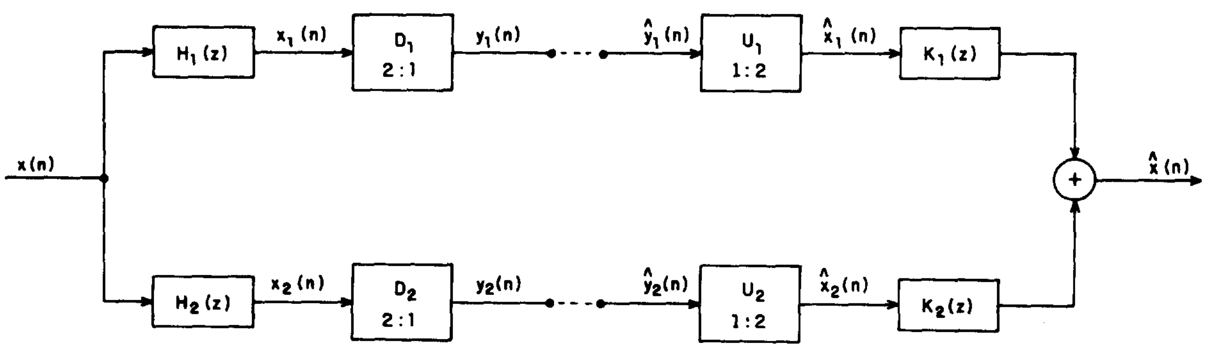

Fig. 1. Block diagram of a two-band subband coding system. The upper branch shows the low-pass signal and the lower branch the high-pass signal.

Recently, Galand and Nussbaumer [3] have observed that the number of multiplications and additions in implementing the filtering operation can be reduced significantly if the length M of the half-band filter $H(z)$ can be made odd. They have suggested a QMF structure which allows the use of an odd-length half-band filter. In this structure, a one-sample delay is introduced in the high-pass filter at the transmitter and a one-sample delay in the low-pass filter at the receiver. Though this QMF structure allows the use of an odd-length half-band filter, it can be easily shown that it increases the group delay to M samples.

In the present study, we derive a number of reduced group delay structures which allow the use of even- as well as odd-length half-band filters. However, before doing this, we want to note that the two QMF structures described earlier (one permitting an even-length filter [2] and the other an odd-length filter [3]) retain only even-numbered samples at the down-sampling stages D_1 and D_2, and put received samples at the even-numbered places at the up-sampling stages U_1 and U_2. In other words, these two structures use the same polarity (even) at all the four stages D_1, U_1, D_2 and U_2.

In the present paper, we argue that it is not necessary to use the same polarity at all the four stages. If we allow any polarity (even or odd) at the four stages, we get 16 different combinations. In the present paper, we consider for each of these 16 combinations the following three different cases:

(1) $H_1(z) = H(z)$ and $H_2(z) = H(-z)$.

(2) $H_1(z) = H(z)$ and $H_2(z) = z^{-1}H(-z)$.

(3) $H_1(z) = z^{-1}H(z)$ and $H_2(z) = H(-z)$.

These 16 combinations with 3 cases result in 48 different QMF structures. Here we derive these 48 QMF structures in causal as well as in noncausal forms. The noncausal QMF structures have the limitation that they can be applied to only those subband coders where speech is processed in a block-wise fashion. The causal QMF structures do not have this limitation. They can be used with any type of subband coder. The causal and noncausal QMF structures are described below.

2.1. Causal QMF structures

The 48 causal QMF structures are listed in Table 1. (Details regarding the derivation of this table are given in Appendix A.)

The two QMF structures described earlier (one allowing the use of an even-length half-band filter with a group delay of $(M-1)$ samples [2] and the other an odd-length half-band filter with a group delay of M samples [3]) are shown in Table 1 as the structures numbered 46 and 47. We can see from this table that there are a number of QMF structures which allow the use of an even- or an odd-length filter. Also, some of the odd-length QMF structures permit a reduction in group delay. For example, the structure numbered 10 can be used with an odd-length FIR filter resulting in a group delay of only $(M-1)$ samples; while Galand and Nussbaumer's structure [3] (listed at the 47th row of Table 1) allows for an odd-length FIR filter with a group delay of M samples. This means that we can use an odd-length filter in a QMF structure for reducing the number of multiplications and additions [3] without increasing the group delay.

2.2. Noncausal QMF structures

As mentioned earlier, the noncausal QMF structures have the limitation that they can be used only with those subband coders which process speech in a block mode. These subband coders (operating in block mode) have the disadvantage that they introduce an additional delay of N samples (where N is the block length). However, if a particular subband coder permits processing of the speech signal in a block mode, the noncausal QMF structures may be useful for this coder. In this subsection, we consider noncausal QMF structures.

Details regarding the derivation of 48 noncausal QMF structures are given in Appendix B. Since only 21 of these structures are different from the corresponding causal QMF structures (shown in Table 1), we list here in Table 2 only these 21 noncausal QMF structures.

Table 1
Causal QMF structures

Structure number	D_1	U_1	D_2	U_2	$H_1(z)$	$H_2(z)$	$K_1(z)$	$K_2(z)$	Filter length	Group delay
1	odd	odd	odd	odd	$H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	$M-1$
2	odd	odd	odd	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	M
3	odd	odd	odd	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	M
4	odd	odd	odd	even	$H(z)$	$H(-z)$	$-z^{-1}H(z)$	$H(-z)$	even	M
5	odd	odd	odd	even	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$H(-z)$	odd	$M+1$
6	odd	odd	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	M
7	odd	odd	even	odd	$H(z)$	$H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	M
8	odd	odd	even	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$-H(-z)$	even	$M+1$
9	odd	odd	even	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	M
10	odd	odd	even	even	$H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	$M-1$
11	odd	odd	even	even	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	M
12	odd	odd	even	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	M
13	odd	even	odd	odd	$H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	M
14	odd	even	odd	odd	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$H(-z)$	odd	M
15	odd	even	odd	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-2}H(-z)$	odd	$M+1$
16	odd	even	odd	even	$H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	M
17	odd	even	odd	even	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	$M+1$
18	odd	even	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	$M+1$
19	odd	even	even	odd	$H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	M
20	odd	even	even	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	$M+1$
21	odd	even	even	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	$M+1$
22	odd	even	even	even	$H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	M
23	odd	even	even	even	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$-H(-z)$	even	M
24	odd	even	even	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-2}H(-z)$	even	$M+1$
25	even	odd	odd	odd	$H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	M
26	even	odd	odd	odd	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$-H(-z)$	even	M
27	even	odd	odd	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-2}H(-z)$	even	$M+1$
28	even	odd	odd	even	$H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	M
29	even	odd	odd	even	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	$M+1$
30	even	odd	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	$M+1$
31	even	odd	even	odd	$H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	M
32	even	odd	even	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	$M+1$
33	even	odd	even	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	$M+1$
34	even	odd	even	even	$H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	M
35	even	odd	even	even	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$H(-z)$	odd	M
36	even	odd	even	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-2}H(-z)$	odd	$M+1$
37	even	even	odd	odd	$H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	$M-1$
38	even	even	odd	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	M
39	even	even	odd	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	M
40	even	even	odd	even	$H(z)$	$H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	M
41	even	even	odd	even	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$-H(-z)$	even	$M+1$
42	even	even	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$H(-z)$	even	M
43	even	even	even	odd	$H(z)$	$H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	M
44	even	even	even	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$H(-z)$	odd	$M+1$
45	even	even	even	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	M
46	even	even	even	even	$H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	$M-1$
47	even	even	even	even	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	M
48	even	even	even	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	M

It can be seen from Table 2 that like the causal QMF structures, the noncausal QMF structures can use even- as well as odd-length FIR filters. It is possible here also to reduce the group delay to $(M-1)$ samples for the odd-length QMF's. In addition, the noncausal QMF structures have an interesting example which uses even-length FIR filter and reduces the group delay to $(M-2)$ samples. This example is shown in Table 2 as the 16th QMF structure.

3. Conclusions

In this paper, we have derived new causal and noncausal QMF structures based on even- as well as odd-length half-band FIR filters. It is shown that for noncausal structures we can reduce the group delay to $(M-2)$ samples for the even-length QMF's and to $(M-1)$ samples for the odd-length QMF's. For causal structures, the minimum group delay is $(M-1)$ samples for even- as well as odd-length QMF's.

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Appendix A. Derivation of causal QMF structures

Here, we illustrate the procedure for deriving the causal QMF structures listed in Table 1. For this, consider the two-band subband coding system shown in Fig. 1. As mentioned earlier, we assume here the channel to be ideal; i.e., $\hat{y}_1(n)=y_1(n)$ and $\hat{y}_2(n)=y_2(n)$. Consider the low-pass branch of the subband coding system.

We first consider the down-sampler D_1. It can either retain even-numbered samples (and discard

Table 2
Noncausal QMF structures

Structure number	D_1	U_1	D_2	U_2	$H_1(z)$	$H_2(z)$	$K_1(z)$	$K_2(z)$	Filter length	Group delay
4	odd	odd	odd	even	$H(z)$	$H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	$M-1$
5	odd	odd	odd	even	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$H(-z)$	odd	$M-1$
6	odd	odd	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-2}H(-z)$	odd	M
13	odd	even	odd	odd	$H(z)$	$H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	$M-1$
14	odd	even	odd	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$H(-z)$	odd	M
15	odd	even	odd	odd	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$H(-z)$	odd	$M-1$
16	odd	even	odd	even	$H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	$M-2$
17	odd	even	odd	even	$H(z)$	$z^{-1}H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	$M-1$
18	odd	even	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	$M-1$
19	odd	even	even	odd	$H(z)$	$H(-z)$	$z^{-2}H(z)$	$H(-z)$	odd	M
20	odd	even	even	odd	$H(z)$	$z^{-1}H(-z)$	$z^{-3}H(z)$	$-H(-z)$	even	$M+1$
21	odd	even	even	odd	$z^{-1}H(z)$	$H(-z)$	$z^{-1}H(z)$	$-H(-z)$	even	M
22	odd	even	even	even	$H(z)$	$H(-z)$	$z^{-1}H(z)$	$H(-z)$	odd	$M-1$
23	odd	even	even	even	$H(z)$	$z^{-1}H(-z)$	$z^{-2}H(z)$	$-H(-z)$	even	M
24	odd	even	even	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-H(-z)$	even	$M-1$
28	even	odd	odd	even	$H(z)$	$H(-z)$	$H(z)$	$z^{-2}H(-z)$	odd	M
29	even	odd	odd	even	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$-z^{-1}H(-z)$	even	M
30	even	odd	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-3}H(-z)$	even	$M+1$
40	even	even	odd	even	$H(z)$	$H(-z)$	$H(z)$	$z^{-1}H(-z)$	odd	$M-1$
41	even	even	odd	even	$H(z)$	$z^{-1}H(-z)$	$H(z)$	$-H(-z)$	even	$M-1$
42	even	even	odd	even	$z^{-1}H(z)$	$H(-z)$	$H(z)$	$-z^{-2}H(-z)$	even	M

odd-numbered samples) or retain odd-numbered samples (and discard even-numbered samples). We write $D_1 = \text{even}$ when the even-numbered samples are retained; and $D_1 = \text{odd}$ when the odd-numbered samples are retained.

Next, we consider the up-sampler U_1. It can either put the received samples at even-numbered places (and zeros at odd-numbered places) or put the received samples at odd-numbered places (and zeros at even-numbered places). We denote $U_1 = \text{even}$ when the received samples are put at the even-numbered places; and $U_1 = \text{odd}$ when these samples are put at the odd-numbered places.

The down-sampler D_1 and the up-sampler U_1 can be combined in four different ways as follows:

(1) $D_1 = \text{even}$, $U_1 = \text{even}$,
(2) $D_1 = \text{even}$, $U_1 = \text{odd}$,
(3) $D_1 = \text{odd}$, $U_1 = \text{even}$, and
(4) $D_1 = \text{odd}$, $U_1 = \text{odd}$.

The signals $\hat{x}_1(n)$ resulting from these four combinations of down-sampler D_1 and up-sampler U_1 are shown in Fig. A.1. The z-transform $\hat{X}_1(z)$ of the signal $\hat{x}_1(n)$ can be written for each of these four combinations, using Fig. A.1, as follows:

Combination 1: $D_1 = \text{even}$ and $U_1 = \text{even}$:

$$\hat{X}_1(z) = x_1(0) + x_1(2)z^{-2} + x_1(4)z^{-4} + \cdots$$
$$= \tfrac{1}{2}[X_1(z) + X_1(-z)].$$

Combination 2: $D_1 = \text{even}$ and $U_1 = \text{odd}$:

$$\hat{X}_1(z) = x_1(0)z^{-1} + x_1(2)z^{-3} + x_1(4)z^{-5} + \cdots$$
$$= \tfrac{1}{2}z^{-1}[X_1(z) + X_1(-z)].$$

Combination 3: $D_1 = \text{odd}$ and $U_1 = \text{even}$:

$$\hat{X}_1(z) = x_1(1)z^{-2} + x_1(3)z^{-4} + x_1(5)z^{-6} + \cdots$$
$$= \tfrac{1}{2}z^{-1}[X_1(z) - X_1(-z)].$$

Combination 4: $D_1 = \text{odd}$ and $U_1 = \text{odd}$:

$$\hat{X}_1(z) = x_1(1)z^{-1} + x_1(3)z^{-3} + x_1(5)z^{-5} + \cdots$$
$$= \tfrac{1}{2}[X_1(z) - X_1(-z)].$$

In a similar manner, the z-transform $\hat{X}_2(z)$ of the signal $\hat{x}_2(n)$ can be written for each of the four combinations of D_2 and U_2. These z-transforms are listed in Table A.1.

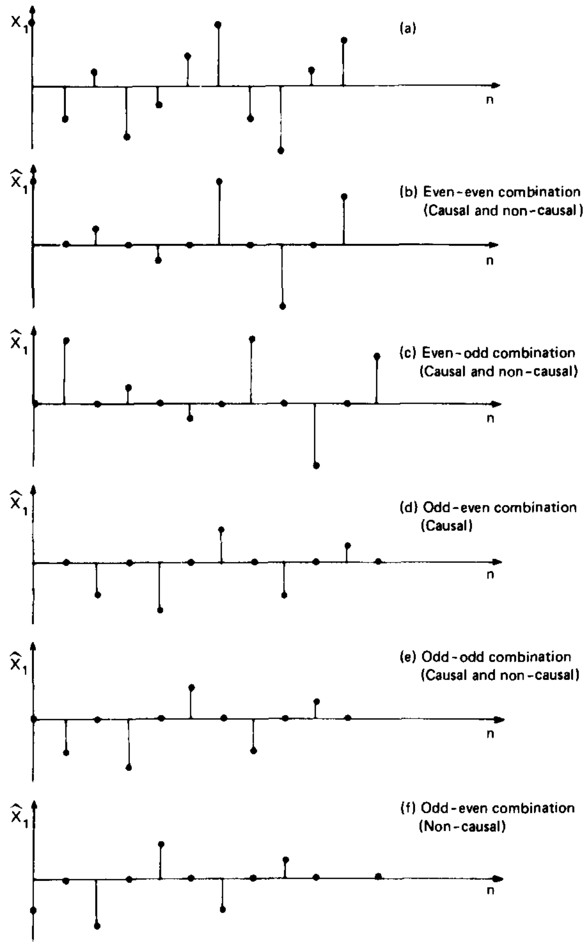

Fig. A.1. Illustration of different combinations of down-sampling and up-sampling operations for the causal and noncausal structures.

Using this table, we can write down the z-transform $\hat{X}(z)$ of the output signal $\hat{x}(n)$ in terms of z-transform $X(z)$ of the input signal $x(n)$ for any possible combination of D_1, U_1, D_2 and U_2. For example, let us consider the combination: $D_1 = \text{even}$, $U_1 = \text{odd}$, $D_2 = \text{even}$ and $U_2 = \text{even}$. For this combination we can write $\hat{X}(z)$ directly using Table A.1 and Fig. 1 as follows:

$$\hat{X}(z) = \tfrac{1}{2}z^{-1}\{X(z)H_1(z) + X(-z)H_1(-z)\}K_1(z)$$
$$+ \tfrac{1}{2}\{X(z)H_2(z) + X(-z)H_2(-z)\}K_2(z)$$
$$= \tfrac{1}{2}\{z^{-1}H_1(z)K_1(z) + H_2(z)K_2(z)\}X(z)$$
$$+ \tfrac{1}{2}\{z^{-1}H_1(-z)K_1(z)$$
$$+ H_2(-z)K_2(z)\}X(-z).$$

Table A.1
The z-transform $\hat{X}_i(z)$ of the up-sampled signal for the *causal* QMF structures for the four different combinations (here, $i=1$ corresponds to the low-pass signal and $i=2$ to the high-pass signal)

D_i	U_i	$\hat{X}_i(z)$
even	even	$\frac{1}{2}[X_i(z)+X_i(-z)]$
even	odd	$\frac{1}{2}z^{-1}[X_i(z)+X_i(-z)]$
odd	even	$\frac{1}{2}z^{-1}[X_i(z)-X_i(-z)]$
odd	odd	$\frac{1}{2}[X_i(z)-X_i(-z)]$

Here, the second term represents the aliasing distortion introduced at the down-sampling and up-sampling stages.

Consider the case where $H_1(z)=H(z)$ and $H_2(z)=H(-z)$. The aliasing term in the above equation vanishes only when we select $K_1(z)=H(z)$ and $K_2(z)=-z^{-1}H(-z)$. The z-transform $\hat{X}(z)$ of the output signal is then given by

$$\hat{X}(z)=\tfrac{1}{2}z^{-1}[H^2(z)-H^2(-z)]X(z).$$

This equation reveals two points. Firstly, the output signal $\hat{x}(n)$ is delayed with respect to the input signal $x(n)$ by M samples. Secondly, the output signal will be a replica of the input signal (except for delay and scale factor) only when the length M of the half-band FIR filter $H(z)$ is even. This QMF structure (corresponding to $D_1=\text{even}$, $U_1=\text{odd}$, $D_2=\text{even}$, $U_2=\text{even}$, $H_1(z)=H(z)$, $H_2(z)=H(-z)$, $K_1(z)=H(z)$ and $K_2(z)=-z^{-1}H(-z)$) is shown in Table 1 at the 34th row.

In a similar manner, all the 48 causal QMF structures listed in Table 1 can be derived.

Appendix B. Derivation of noncausal QMF structures

Here, we illustrate the procedure for deriving the noncausal QMF structures listed in Table 2. For this, we refer to Fig. A.1 where the signals $\hat{x}_1(n)$ resulting from four different combinations of down-sampler D_1 and up-sampler U_1 are shown for both causal and noncausal structures. From this figure, it is clear that the signals $\hat{x}_1(n)$ for the causal and noncausal structures are the same for the three combinations (even–even, even–odd and odd–odd) and different for the odd–even combination. Because of this, the z-transforms of the up-sampled signals resulting from the odd–even combination only are different for the causal and noncausal combinations.

The z-transforms of the up-sampled signals resulting from the four different combinations can be easily computed for the noncausal structures from Fig. A.1. These are listed in Table B.1.

Table B.1
The z-transform $\hat{X}_i(z)$ of the up-sampled signal for the *noncausal* QMF structures for the four different combinations (here, $i=1$ corresponds to the low-pass signal and $i=2$ to the high-pass signal)

D_i	U_i	$\hat{X}_i(z)$
even	even	$\frac{1}{2}[X_i(z)+X_i(-z)]$
even	odd	$\frac{1}{2}z^{-1}[X_i(z)+X_i(-z)]$
odd	even	$\frac{1}{2}z[X_i(z)-X_i(-z)]$
odd	odd	$\frac{1}{2}[X_i(z)-X_i(-z)]$

Using this table, the 48 noncausal QMF structures can be easily derived. However, 27 of these noncausal structures (namely, 1–3, 7–12, 25–27, 31–39 and 43–48) are identical to the corresponding causal structures (listed in Table 1). Because of this, we list here in Table 2 only 21 remaining noncausal structures which are different from the corresponding causal structures.

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