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SHORT COMMUNICATION

A NOISE-COMPENSATED LONG CORRELATION MATCHING METHOD FOR AR SPECTRAL ESTIMATION OF NOISY SIGNALS

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Abstract. A noise-compensated long correlation matching (NCLCM) method is proposed for autoregressive (AR) spectral estimation of the noisy AR signals. This method first computes the AR parameters from the high-order Yule-Walker equations. Next, it employs these AR parameters and uses the low-order Yule-Walker equations to compensate the zeroth autocorrelation coefficient for the additive white noise. Finally, it solves the low- as well as high-order Yule-Walker equations in a least-squares sense to determine the AR parameters. It is shown that for the noisy AR signals the NCLCM method performs better than the conventional Burg method and the high-order Yule-Walker method

Zusammenfassung. Zur autoregressiven (AR) Spektralschätzung verrauschter AR-Signale wird ein Verfahren vorgeschlagen, das eine Rauschkompensation und eine Anpassung an die Langzeitkorrelation beinhaltet. Hierbei werden zunächst die AR-Parameter aus einem Yule-Walker Gleichungssystem hoher Ordnung berechnet. Danach werden dieser Parameter und ein Yule-Walker Gleichungssystem niedriger Ordnung benützt, um den Einfluß des additiven weißen Rauschens beim nullten Autokorrelationskoeffizienten zu kompensieren. Schließlich werden beide Gleichungssysteme im Sinne eines minimalen Fehlerquadrats gelöst, um die endgültigen AR-Parameter zu erhalten. Es wird gezeigt, daß diese Methode für verrauschte AR-Signale besser funktioniert als das übliche Vorgehen nach Burg oder die Verwendung eines hochgradigen Yule-Walker Ansatzes allein.

Résumé. Une méthode d'adaptation par corrélation longue et compensée en bruit est proposée pour l'estimation spectrale autorégressive (AR) de signaux bruités AR. Cette méthode calcule d'abord les paramètres AR à partir des équations d'ordre supérieur de Yule-Walker. Ensuite, elle utilise ces paramètres AR et met en oeuvre des équations d'ordre inférieur de Yule-Walker pour compenser le coefficient de corrélation d'indice zéro pour du bruit blanc additif. Finalement, elle résoud les équations d'ordre supérieur aussi bien que d'ordre inférieur de Yule-Walker au sens des moindres carrés pour déterminer les paramètres AR. Il est montré que pour un signal AR bruité, cette méthode est meilleure que la méthode conventionnelle de Burg et la méthode de Yule-Walker d'ordre supérieur.

Keywords: Spectral estimation, AR modeling, noisy signal, Yule-Walker equations.

1. Introduction

For signals following the autoregressive (AR) model, the conventional AR spectral estimation methods (such as the forward-backward linear prediction method and the Burg method) perform reasonably well [1]. But, when these signals are corrupted by the addition of white noise, the AR model is no more valid and, as a consequence, the performance of the conventional methods is poor for noisy signals [2]. A number of methods have been proposed in the literature to solve the problem of AR spectral estimation for noisy signals [3-5]. These methods avoid the use of zeroth autocorrelation coefficient by employing the high-order Yule-Walker equations for estimating the AR parameters. Since these methods do not use the low-order Yule-Walker equations, their statistical performance is not very satisfactory [6].

In the present paper, we propose a new method, called the noise-compensated long correlation matching (NCLCM) method, for AR spectral estimation of noisy signals. This method compensates the zeroth autocorrelation coefficient for the additive white noise and uses both low- and high-order Yule-Walker equations for estimating AR parameters. It is shown that this method is statistically more efficient than the method using the high-order Yule-Walker equations only. Furthermore, since this method uses more Yule-Walker equations than the HOYW method, it is numerically more stable.

2. The NCLCM method

The NCLCM method computes the parameters $\{a_i, i=1,2,\ldots,p\}$ of the pth order AR model by using the following four steps:

Step 1. Compute the unbiased estimate of the $(p+q)$ autocorrelation coefficients $\{R(i), i=1,2,\ldots,p+q\}$ from the observed noisy signal $\{y(n), n=1,2,\ldots,N\}$, where $q>p$.

Step 2. With the knowledge of $(p+q)$ autocorrelation coefficients $\{R(i), i=1,2,\ldots,p+q\}$ from Step 1, compute (in a least-squares sense) the AR parameters $\{a_i\}$ from an overdetermined set of q high-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(i-k) = -R(i),$$

$$i=p+1, p+2, \ldots, p+q.$$

Step 3. With the knowledge of AR parameters from Step 2 and p autocorrelation coefficients $\{R(i), i=1,2,\ldots,p\}$ from Step 1, compute (in a least-squares sense) the zeroth autocorrelation coefficient $R(0)$ from the p low-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(|i-k|) = -R(i), \quad i=1,2,\ldots,p.$$

Step 4. With the knowledge of zeroth autocorrelation coefficient $R(0)$ from Step 3 and $(p+q)$ autocorrelation coefficients $\{R(i), i=1,2,\ldots,p+q\}$ from Step 1, compute (in a least-squares sense) the desired AR parameters $\{a_i\}$ from an overdetermined set of the following $(p+q)$ Yule-Walker equations which include both low- and high-order Yule-Walker equations:

$$\sum_{k=1}^{p} a_k R(|i-k|) = -R(i), \quad i=1,2,\ldots,p+q.$$

3. Results

In this section, we study the NCLCM method for the noisy AR signals. In order to put this method in a proper perspective, we compare here its performance with that of the conventional Burg method and the HOYW method.

Statistical bias and variance in estimating the AR parameters are used here as criteria for performance evaluation. Since there is always a trade off involved between the bias and the variance, we combine the bias and the variance and use the root-mean-square (RMS) error as a measure of the overall performance of the spectral estimation method. The RMS error is defined as:

$$\text{RMS error} = \sqrt{\text{Bias}^2 + \text{Variance}}.$$

Following Reddi [7], we also define the normalized total-mean-square (NTMS) error as a measure of spectral estimation performance. This measure combines the RMS errors of the individual AR parameters as follows:

$$\hat{E} = 10\log_{10}\left(\sum_{i=1}^{p} \Delta a_i^2 \Big/ \sum_{i=1}^{p} a_i^2\right), \quad \text{(in dB)}$$

where Δa_i is the RMS error in estimating the ith AR parameter.

For illustration, we select here an example of the AR process which has been commonly used in the literature [2, 4]. In this example, the signal is generated from a fourth-order AR system (with parameters $a_1=-2.7607$, $a_2=3.8106$, $a_3=-2.6535$ and $a_4=0.9238$), driven by zero-mean unit-variance white Gaussian noise process. Here the

AR signal consists of two narrow-band peaks in its spectrum.

We generate 100 independent realizations of the fourth-order AR process and use $N = 40$ data points during steady-state to compute AR parameters for each of these realizations. Different spectral estimation methods are used here with $p = 4$ and $p + q = \frac{1}{2}N$. Statistical bias, variance and root-mean-square (RMS) error in estimating AR parameters are computed by ensemble-averaging over these 100 realizations. Results are listed in Table 1 for the Burg, the HOYW and the NCLCM methods. It can be seen from this table that the NCLCM method results in better performance than the Burg and the HOYW methods. In terms of NTMS error, it shows an advantage of 7.8 dB over the Burg method and 0.9 dB over the HOYW method. Also, it can be seen that the Burg method results in smaller variance than the HOYW and the NCLCM methods, but it has too much of bias and, hence, its overall performance is significantly poorer than the other two methods.

In order to see the performance of these methods for long data records, we study them for the noisy AR signal (with SNR = 20 dB) for $N = 140$ data points. Results are shown in Table 2. It can be seen from Table 2 that the NCLCM method is only marginally superior to the HOYW method, but it is significantly better than the Burg method. Also, the variance in estimating the AR parameters is comparable for the three methods, while the bias from the Burg method is significantly more than that from the other two methods.

Next, we study the effect of data-record length on the performance of the three methods (Burg, HOYW and NCLCM). For this, we take SNR = 20 dB and compute the value of the NTMS error for different data-record lengths. Results are shown in Fig. 1. It can be observed from this figure that the HOYW and the NCLCM methods show improvement with record length, while the Burg method shows no such improvement. We also notice that the NCLCM method results in better

Table 1

Statistical performance of different methods in estimating the AR parameters $\{a_1, a_2, a_3, a_4\}$ for the noisy AR signal at SNR = 20 dB with $N = 40$ data points

	a_1	a_2	a_3	a_4
True values	−2.7607	3.8106	−2.6535	0.9238
Bias Burg	1.6418	−3.2937	2.9936	−0.9751
Bias MOYW	0.5130	−1.0876	1.0173	−0.3662
Bias NCLCM	0.4371	−0.9417	0.8908	−0.3284
Variance Burg	0.0387	0.0745	0.0599	0.0243
Variance HOYW	0.2430	0.9823	0.8683	0.1239
Variance NCLCM	0.2066	0.8679	0.7865	0.1143
RMS error Burg	1.6535	3.3050	3.0036	0.9874
RMS error HOYW	0.7114	1.4714	1.3795	0.5080
RMS error NCLCM	0.6306	1.3247	1.2570	0.4713

Table 2

Statistical performance of different methods in estimating the AR parameters $\{a_1, a_2, a_3, a_4\}$ for the noisy AR signal at SNR = 20 dB with $N = 140$ data points

	a_1	a_2	a_3	a_4
True values	−2.7607	3.8106	−2.6535	0.9238
Bias Burg	1.6273	−3.3154	3.0419	−1.0190
Bias HOYW	0.0441	−0.1063	0.1078	−0.0450
Bias NCLCM	0.0422	−0.1032	0.1047	−0.0441
Variance Burg	0.0081	0.0179	0.0120	0.0078
Variance HOYW	0.0064	0.0287	0.0265	0.0043
Variance NCLCM	0.0062	0.0277	0.0253	0.0040
RMS error Burg	1.6298	3.3181	3.0439	1.0228
RMS error HOYW	0.0915	0.2000	0.1951	0.0796
RMS error NCLCM	0.0893	0.1958	0.1905	0.0773

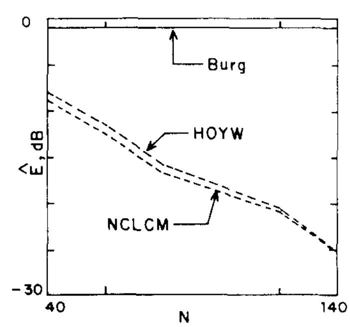

Fig. 1. Normalized total-mean-square error as a function of record length, SNR = 20 dB.

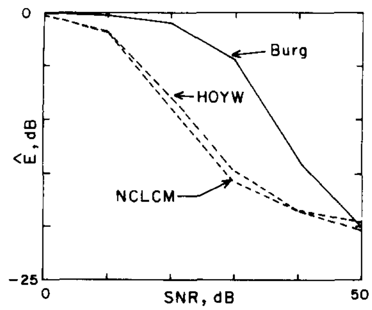

Fig. 2. Normalized total-mean-square error as a function of SNR, $N = 40$.

performance than the HOYW method for all record lengths (and more so for short records).

So far, we have studied the Burg, the HOYW and the NCLCM methods for the noisy signals having a fixed SNR value. Now, we study these methods for different SNR values. For this, we take $N = 40$ and study the NTMS error as a function of SNR. Results are shown in Fig. 2. We can see from this figure that the performance of each of the three methods improves with SNR. For low SNR (0–10 dB) and high SNR (40–50 dB) values, the three methods are comparable in performance. For medium SNR values (20–30 dB), the NCLCM method shows advantage over the other two methods.

4. Conclusions

In the present paper, the NCLCM method has been proposed for AR spectral estimation of the noisy signals. We have studied the performance of this method for different data-record lengths and different SNR values. It has been shown that for medium SNR values (20–30 dB) the NCLCM method performs better than the Burg and the HOYW methods for all record lengths. But, for low SNR (0–10 dB) and high SNR (40–50 dB) values the three methods are comparable in performance.

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